# A Dipolar Bose-Einstein Condensate in a one-dimensional Optical Lattice 

Diplomarbeit<br>vorgelegt von<br>Holger Kadau



Hauptberichter: Prof. Dr. Tilman Pfau
Mitberichterin: Prof. Dr. Gisela Schütz
5. Physikalisches Institut

Universität Stuttgart
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## 1. Introduction

In the last years, it has been shown that ultracold atomic gases provide an 'ideal' environment for studying many-body quantum systems usually encountered in condensed matter physics [1, 2]. Both the internal and the external parameters can be controlled with high precision and the inter-particle contact interaction can be tuned almost at will [3]. Since 1995 [4-6] quantum gases with isotropic short-range contact interaction have been extensively studied. For ultracold gases, the contact interaction strength is characterized by one single parameter, the s-wave scattering length $a$. In 2005 the experimental realization of chromium Bose-Einstein condensates (BECs) [7] opened the door to the investigation of strong inter-atomic magnetic dipole-dipole interaction in degenerate quantum gases. In strong contrast with the contact interaction, the dipolar interaction is anisotropic and long-range. The dipolar interaction in ultracold gases has attracted a lot of attention, both experimentally and theoretically $[8,9]$.

Experimentally, strong dipolar effects in cold quantum gases have been shown in systems of chromium BECs [10-12], ultracold fermionic heteronuclear molecules [13, 14] and BECs with Rydberg atoms [15]. In alkali gases weak dipolar effects have been observed, when tuning the contact interaction to vanishing interaction strength [16-18]. Recently, progress has been made towards the realization of ultracold gases of dysprosium atoms, which have the highest magnetic moment of all stable elements [19].

Specifically for chromium, the experiments have concentrated on the anisotropic character of the dipolar interaction. In particular the following effects have been observed: the dipolar strength was enhanced by decreasing the contact interaction and its effect on the expansion dynamics was explored [10]. For further decreasing the contact interaction, the dipolar BEC shows a strong trap geometry dependent instability threshold [11]. It was shown for certain trap geometries, the dipolar BEC collapses in a d-wave symmetry [20]. Another effect of the anisotropic dipolar interaction is the coupling of the spin degree of freedom to the orbital momentum, hence, dipolar collisions do not conserve the magnetic substates [21]. This effect has been used to successfully cool a chromium cloud via demagnetization cooling [22]. Moreover, the dipolar interaction influences the collective mode frequencies of a BEC [23].

Theoretically, unique self-organized structures have been predicted, such as density modulated ground states [24, 25], where the maximum density is not centered in the external trapping potential, or supersolid phases in optical lattices [26, 27]. These effects are closely related to the non-monotonically increasing excitation spectrum of a dipolar gas, which displays a roton minimum [28] similar to the one observed in superfluid helium [29]. Due to the anisotropy of the dipolar interaction, these effects are predicted only in
highly oblate trapping geometries. Before the start of this diploma thesis, this range of parameters was in an experimentally unreached regime and it was unclear whether we could reach this regime or if the dipolar BEC would be unstable.

In earlier experiments in our group, the static properties of a single dipolar BEC were measured. It was shown that the stability of such a system depends strongly on the geometry of the trap holding it [11]. In a prolate trap, the dipolar interaction is mainly attractive and leads to a destabilization of the BEC. The dipolar BEC becomes then unstable even for repulsive contact interaction. In contrary, for a highly oblate trap, the dipolar interaction is mainly repulsive and has a stabilizing effect. Therefore, it is in principle possible to realize stable dipolar BECs with destabilizing attractive contact interaction in highly oblate traps. The region with repulsive contact interaction was investigated and a purely dipolar interacting BEC was generated. However, the regime with attractive contact interaction could not be reached.

During the time of this diploma thesis, we have indeed reached experimentally this interesting regime, where the above mentioned unique dipolar effects are expected. This is done by slicing a chromium BEC using a one-dimensional optical lattice. Effectively for sufficiently deep lattices, the system can be seen as a stack of highly oblate dipolar condensates, each BEC localized on one lattice site. We investigate the static properties of a ${ }^{52} \mathrm{Cr}$ BEC in the one-dimensional lattice and we stabilize a dipolar BEC for attractive contact interaction up to scattering lengths of $a=-17$ Bohr radii. This work does not only complete the earlier stability measurements, as we reach highly oblate traps, but also realizes a system with long-range dipolar interactions between the BECs (inter-site interactions). We observe a strong influence of the inter-site interactions on the stability threshold. Studying the stability of this system is a crucial step towards the realization of novel quantum phases. In addition in such coupled systems, the theoretically predicted effects are even enhanced [30-32], due to the long-range dipolar interactions between the dipolar BECs. A preprint with the above mentioned results is already available [33].

## This thesis is organized as follows:

In chapter 2 we give the necessary theoretical elements required to describe a dipolar BEC. After an overview on Bose-Einstein condensation, we describe the two inter-atomic interactions present in the system, namely the contact interaction and the dipolar interaction. We see that the scattering length $a$ can be tuned by using a so-called Feshbach resonance [3]. With this knowledge various properties of a dipolar BEC are discussed, as for example its expansion and its stability conditions.

In the following chapter, we show how to realize a chromium BEC with strong dipolar effects. We demonstrate how we decrease the contact interaction strength using a magnetic Feshbach resonance to enhance the effects of the dipoles. This implies to know precisely the scattering length and we present our method to calibrate it.

In chapter 4 we describe the one-dimensional optical lattice that we use to confine
the chromium BEC in highly oblate traps. Its properties can be described by a single parameter: the lattice depth $V_{0}$. For the calibration of this important parameter, we use two independent experimental methods: the interference patterns obtained after the BEC expansion and the diffraction of a BEC from a pulsed optical lattice.

In the last chapter we investigate the stability of a chromium BEC in the one-dimensional optical lattice. We show that we generate a stable dipolar BEC even with attractive contact interaction. The destabilizing attractive contact interaction is stabilized by the mainly repulsive dipolar interaction. Additionally we observe a destabilizing effect of the attractive inter-site dipolar interactions. Finally, to get deeper insight in the effects of the inter-site interactions, we develop a model to describe the destabilizing effect of the interactions between spatially separated dipolar BECs.

The appendix gives additional information on the two-frequency acousto-optical modulator drivers [34] that were renewed during this diploma thesis. Furthermore, three automatic evaluation procedures were programmed and are described in A.2. All of them were necessary for the evaluation of the measurements in chapter 5 .

## 2. Dipolar Bose-Einstein Condensates

This chapter introduces the basic theoretical elements necessary to describe a dipolar Bose-Einstein condensate (BEC). We first discuss the case of an ultracold atomic sample that can undergo a phase transition to a BEC below a critical temperature $T_{c}$. These condensates have non-negligible inter-atomic interactions: the contact interaction and the dipolar interaction.

In section 2.2 we describe the case of an almost purely contact interacting gas with negligible dipolar effects, like alkali atoms ${ }^{1}$. Contact interaction is short-range and isotropic. For a many-body system, it is well described with a mean field ansatz. Due to the large magnetic moment of chromium ${ }^{2}$ the anisotropic long-range dipolar interaction is not negligible. The interplay between contact and dipolar interactions opens the way to new ultracold atomic physics. In section 2.3 we discuss the mean field description for the dipolar interaction, which can describe various properties of a dipolar BEC. For example its expansion and its stability are discussed in detail in sections 2.3.3 and 2.3.4.

### 2.1. Bose-Einstein Condensation

In 1995 the first BECs have been produced in vapors of atomic rubidium, sodium and lithium [4-6]. Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman received the Nobel Prize in Physics 'for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates' in 2001. There is an easy picture for the formation of a BEC: for high temperatures, the atoms behave 'classically', their thermal de Broglie wavelength (see section 2.1.1 for the definition) being negligible. For lower temperatures, the de Broglie wavelength becomes comparable to the inter-atomic distance and the single wave packets start to overlap until at zero temperature we obtain a pure BEC with all the atoms in the ground state of the system (see fig. 2.1).

However, this ground state is metastable for atoms as it can decay via 3-body collisions. This collisions create a dimer and a single particle with high kinetic energy, which leads to heating and losses. These 3-body losses strongly depend on the atomic density. Therefore another important point for all following considerations is that ultracold gases are dilute: only binary collisions at low energy are relevant.

[^0]

Figure 2.1.: Phase transition from an ideal gas to a Bose-Einstein condensate. For high temperatures, the system is an ideal gas behaving like 'billiard balls'. For lower temperatures the de Broglie wavelength increases and becomes relevant for the correct description of the system: the atoms now behave like wave packets. At the critical temperature the wave packets start to overlap until at zero temperature they create a giant matter wave. Figure taken from [35].

### 2.1.1. Statistical Phase Transition

This section gives a brief overview over the statistics of a non-interacting Bose gas at ultralow temperatures (see [35-40] for a detailed discussion). We start the section with the statistical of free non-interacting BECs. We then consider the case of a BEC trapped in an external potential.

We first discuss the phase transition from an ideal Bose gas to a BEC in a three dimensional box of volume $V$ and containing $N_{\text {tot }}$ bosons. The particle density is then given by $n=N_{\text {tot }} / V$. We have to define two important quantities:

- the thermal de Broglie wavelength

$$
\begin{equation*}
\lambda_{d B} \equiv \sqrt{\frac{2 \pi \hbar^{2}}{m k_{B} T}} \tag{2.1}
\end{equation*}
$$

- and the phase space density

$$
\begin{equation*}
\mathcal{D}=n \lambda_{d B}^{3} \tag{2.2}
\end{equation*}
$$

with the Boltzmann constant $k_{B}$, the atom mass $m$ and temperature $T$. For an ideal gas, the temperatures $T$ are high and the densities $n$ are low. As a consequence the phase space density is $\mathcal{D} \ll 1$. However, if the temperature is reduced and the density increased, the phase space density will increase. For $\mathcal{D}=1$ the inter-atomic distance is equal to the de Broglie wavelength. For bosons, a macroscopic number of atoms populate the ground state when $\mathcal{D}>\zeta(3 / 2) \approx 2.612$, where $\zeta(s)=\sum_{n=1}^{\infty} n^{-s}$ is the Riemann zeta function. This criterium defines a critical temperature for condensation

$$
\begin{equation*}
k_{B} T_{c}^{\text {free }} \equiv \frac{2 \pi \hbar^{2}}{m}\left(\frac{n}{\zeta(3 / 2)}\right)^{2 / 3} \tag{2.3}
\end{equation*}
$$

In principle, a macroscopically populated ground state is not special, as for $T \rightarrow 0$ absolutely all bosons are in the ground state. However, the critical thermal energy $k_{B} T_{c}^{\text {free }}$ is much higher than the energy difference between harmonic oscillator energy levels [39, chapter 3.3.3] [37, chapter 3.2] and therefore the free energy is discontinuous at $T_{c}^{\text {free }}$, which is characteristic for phase transitions. For the BEC phase transition the fraction of 'condensed atoms' in the ground state, this is the so-called order parameter of the phase transition, is given by [37]

$$
\begin{equation*}
\frac{N^{f r e e}}{N_{\text {tot }}}=1-\left(\frac{T}{T_{c}^{\text {free }}}\right)^{3 / 2} \tag{2.4}
\end{equation*}
$$

where $N_{\text {tot }}$ is the total atom number in the system, $N^{\text {free }}$ the number of free condensed atoms is and $T_{c}^{\text {free }}$ the critical temperature in free space.

In experiments the atoms are not free, but most commonly trapped in external potentials of parabolic shape:

$$
\begin{equation*}
V_{e x t}(\boldsymbol{r})=\frac{m}{2}\left(\omega_{x}^{2} x^{2}+\omega_{y}^{2} y^{2}+\omega_{z}^{2} z^{2}\right) \tag{2.5}
\end{equation*}
$$

with $\omega_{i}$ the trapping frequencies. This additional potential has an effect on the critical temperature [36, chapter II.B][35, chapter 4.5] [37, chapter 10.1]

$$
\begin{equation*}
k_{B} T_{c}=\hbar \bar{\omega}\left(\frac{N_{t o t}}{\zeta(3)}\right)^{1 / 3} \approx 0.94 \hbar \bar{\omega} N_{t o t}^{1 / 3} \tag{2.6}
\end{equation*}
$$

with $\bar{\omega}=\left(\omega_{x} \omega_{y} \omega_{z}\right)^{1 / 3}$ the geometric mean of the trapping frequencies. For a noninteracting harmonically trapped BEC, the density distribution is a Gaussian function [38, chapter 2.3]

$$
\begin{equation*}
n_{h o}(\boldsymbol{r})=\frac{N}{\pi^{3 / 2} l_{x} l_{y} l_{z}} e^{-\frac{x^{2}}{l_{x}^{2}}-\frac{y^{2}}{l_{y}^{2}}-\frac{z^{2}}{l_{z}^{2}}} \tag{2.7}
\end{equation*}
$$

corresponding to the first eigenstate of a harmonic oscillator, with the harmonic oscillator lengths

$$
\begin{equation*}
l_{i}=\sqrt{\frac{\hbar}{m \omega_{i}}} \quad i=(x, y, z) . \tag{2.8}
\end{equation*}
$$

We define a characteristic length scale for the system, i.e. the harmonic oscillator length

$$
\begin{equation*}
a_{h o} \equiv \sqrt{\frac{\hbar}{m \bar{\omega}}} . \tag{2.9}
\end{equation*}
$$

The fraction of condensed atoms in a harmonic trap follows a slightly different behaviour than in free space

$$
\begin{equation*}
\frac{N}{N_{t o t}}=1-\left(\frac{T}{T_{c}}\right)^{3} \tag{2.10}
\end{equation*}
$$

with $N$ the condensed atom number and $T_{c}$ the critical temperature.
For example, let us consider an atomic cloud containing $N_{\text {tot }}=40,000$ chromium atoms. On our experiment, such cloud is typically obtained in traps with mean frequency $\bar{\omega}=2 \pi \cdot(250-700) \mathrm{Hz}$, corresponding harmonic oscillator length $a_{h o}=880-525 \mathrm{~nm}$. In this case, the critical temperature $T_{c}$ ranges typically from 385 to 1080 nK .

### 2.2. Contact interacting Bose-Einstein Condensates

### 2.2.1. Short-range Interactions

In the previous section, we looked at the properties of a non-interacting Bose gas. Even though cold gases are very dilute, their properties are strongly influenced by atomic twobody collisions. The interaction between two atoms is mainly due to the van der Waals interaction $\left(\propto\left(-1 / r^{6}\right)\right.$ ) and the resulting interaction $\left(\propto 1 / r^{12}\right)$ of coulomb and exchange interactions. This results in a complicated molecular potential with a characteristic range $r_{0}$ for the interaction. $r_{0}$ is typically much smaller than the mean inter-particle distance for dilute atomic gases. In addition, the atomic gases that we consider are at very low temperatures. Under those conditions, high energy collisions are suppressed and only s-wave scattering remains. As a consequence, 2 -body collisions within dilute ultracold atomic gases can be treated as hard spheres scattering, which is isotropic and short-range. The interaction potential can be written as

$$
\begin{equation*}
U_{\text {contact }}(\boldsymbol{r}) \equiv g \delta(\boldsymbol{r}) \tag{2.11}
\end{equation*}
$$

with $\delta(\boldsymbol{r})$ the Dirac delta function and $g$ the coupling strength. $g$ describes the interaction strength and is connected to the s-wave scattering length $a$

$$
\begin{equation*}
g \equiv \frac{4 \pi \hbar^{2} a}{m} \tag{2.12}
\end{equation*}
$$

### 2.2.2. Gross-Pitaevskii equation

In the previous section the 2-body short range interaction $U_{\text {contact }}$ was investigated. But a BEC is composed of many atoms and one has to take into account all scattering processes.

Because this is complicated, a theory for many particles is needed. The simplest manybody model is the mean-field model. It maps the $N$ particle ${ }^{3}$ problem onto a single particle problem with an effective potential created by all the other particles. A second quantization description for $N$ particles, the Hamiltonian of the system can be written as

$$
\begin{align*}
\hat{H}= & \int \mathrm{d} \boldsymbol{r} \hat{\Psi}^{\dagger}(\boldsymbol{r}, t)\left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{\text {ext }}(\boldsymbol{r})\right) \hat{\Psi}(\boldsymbol{r}, t) \\
& +\frac{1}{2} \int \mathrm{~d} \boldsymbol{r} \mathrm{~d} \boldsymbol{r}^{\prime} \hat{\Psi}^{\dagger}(\boldsymbol{r}, t) \hat{\Psi}^{\dagger}\left(\boldsymbol{r}^{\prime}, t\right) U_{\text {contact }}\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right) \hat{\Psi}\left(\boldsymbol{r}^{\prime}, t\right) \hat{\Psi}(\boldsymbol{r}, t) \tag{2.13}
\end{align*}
$$

with $\hat{\Psi}(\boldsymbol{r}, t)$ and $\hat{\Psi}^{\dagger}(\boldsymbol{r}, t)$ the boson annihilation and creation field operators. The first term of the Hamiltonian corresponds the kinetic and potential energies and the second term is the contact interaction energy. Using the field operator commutation relations $\left[\hat{\Psi}(\boldsymbol{r}, t), \hat{\Psi}^{\dagger}\left(\boldsymbol{r}^{\prime}, t\right)\right]=\delta\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)$ and $\left[\hat{\Psi}(\boldsymbol{r}, t), \hat{\Psi}\left(\boldsymbol{r}^{\prime}, t\right)\right]=0$, one gets the equation of motion with the Heisenberg equation [36, chapter III.A]

$$
\begin{align*}
i \hbar \frac{\partial}{\partial t} \hat{\Psi}(\boldsymbol{r}, t)=[\hat{\Psi}(\boldsymbol{r}, t), \hat{H}]= & \left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{e x t}(\boldsymbol{r})\right. \\
& \left.+\int \mathrm{d} \boldsymbol{r}^{\prime} \hat{\Psi}^{\dagger}\left(\boldsymbol{r}^{\prime}, t\right) U_{\text {contact }}\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right) \hat{\Psi}\left(\boldsymbol{r}^{\prime}, t\right)\right) \hat{\Psi}(\boldsymbol{r}, t) \tag{2.14}
\end{align*}
$$

The large number of atoms in a BEC allow to replace the field operators by the classical field $\hat{\Psi} \rightarrow \Psi(\boldsymbol{r}, t)$. This direct replacement can also be understood in such a way that the wave function $\Psi(\boldsymbol{r}, t)$ is not changed when removing or adding one particle, meaning that quantum fluctuations are neglected [41, chapter 2.3.2]. Generally, such a replacement is not possible, but for our system, as we are considering cold binary collisions in a dilute gas, the Born approximation is applicable. This means that the scattered wave is much weaker than the initial wave and therefore multiple scattering is negligible. Introducing the pseudo-potential $U_{\text {contact }}$ (equation (2.11)) in equation (2.14) one obtains the wellknown Gross-Pitaevskii equation (GPE)

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \Psi(\boldsymbol{r}, t)=\left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{e x t}(\boldsymbol{r})+g|\Psi(\boldsymbol{r}, t)|^{2}\right) \Psi(\boldsymbol{r}, t) . \tag{2.15}
\end{equation*}
$$

The GPE is a non linear differential equation and thereby not analytically solvable. However, by introducing some simple approximations, one can already get insight on the system ground state.

### 2.2.3. Thomas-Fermi approximation

For large atom numbers $N$, the interaction energy term can become much stronger than the kinetic energy, and the kinetic term in the GPE can be neglected. Such approximation

[^1]is called Thomas-Fermi approximation and allows to obtain a simple equation for a static external potential
\[

$$
\begin{equation*}
\left(V_{e x t}(\boldsymbol{r})+g|\Psi(\boldsymbol{r})|^{2}\right) \Psi(\boldsymbol{r})=\mu \Psi(\boldsymbol{r}), \tag{2.16}
\end{equation*}
$$

\]

where $\mu$ is the chemical potential. The density distribution is given by

$$
\begin{equation*}
n_{T F}(\boldsymbol{r})=|\Psi(\boldsymbol{r})|^{2}=\frac{\mu-V_{e x t}(\boldsymbol{r})}{g} \tag{2.17}
\end{equation*}
$$

and mimics the external trapping potential. Therefore, for a harmonic trap the density distribution of the ground state has an inverted parabolic shape

$$
\begin{equation*}
n_{T F}(\boldsymbol{r})=n_{0} \cdot \max \left[1-\frac{x^{2}}{R_{x}^{2}}-\frac{y^{2}}{R_{y}^{2}}-\frac{z^{2}}{R_{z}^{2}}, 0\right] . \tag{2.18}
\end{equation*}
$$

with $n_{0}$ the density at the center of the cloud and $R_{i}$ the Thomas-Fermi radii

$$
\begin{equation*}
R_{i}=\sqrt{\frac{2 \mu}{m \omega_{i}^{2}}} \quad i=(x, y, z) \tag{2.19}
\end{equation*}
$$

By normalizing the BEC wavefunction to the number of atoms $N\left(\int|\Psi(\boldsymbol{r})|^{2} d \boldsymbol{r}=N\right)$ we obtain the central density $n_{0}=15 N /\left(8 \pi R_{x} R_{y} R_{z}\right)$ and the chemical potential [36]

$$
\begin{equation*}
\mu=\frac{\hbar \bar{\omega}}{2}\left(\frac{15 N a}{a_{h o}}\right)^{2 / 5} \tag{2.20}
\end{equation*}
$$

For a BEC with $N=20,000$ chromium atoms in a trap with mean frequency $\bar{\omega}=$ $2 \pi \cdot(250-700) \mathrm{Hz}$, the typical size of the BEC is around $4-2.6 \mu \mathrm{~m}$ and therefore much larger than the mean harmonic oscillator length $a_{h o}$ calculated in section 2.1.1.

### 2.2.4. Tuning the scattering length with a Feshbach Resonance

As discussed in the previous sections the contact interaction has a strong influence on the BEC properties. It has been shown [3] that it is possible to change the value of the scattering length $a$ by means of a Feshbach resonance. This powerful tool to control short-range interactions in an ultracold gas can be understood in a simple picture: let us consider two molecular potential curves $V_{b g}(R)$ and $V_{c}(r)$, as shown in figure 2.2. The potential $V_{b g}(R)$ describes the interaction of two free atoms with small kinetic energy $E$. This represents the energetically open channel for the collision process. The second potential $V_{c}(r)$ is a closed channel that supports a bound molecular state at the energy level $E_{c}$. This second channel describes e.g. the molecular potential of atoms in different internal magnetic sublevels. If the two channels have different magnetic moments, it is possible to bring the energy levels in resonance with a magnetic field $B$ via the Zeeman shift. This leads to a resonant behaviour of the scattering process, where the scattering


Figure 2.2.: Basic two-channel model for a Feshbach resonance. Open channel $V_{b g}(R)$ (black) with collision energy $E$ and closed channel $V_{c}(R)$ (red) with a bound state at energy $E_{c}$ can be tined in resonance via Zeeman-Shift that leads to a mixing of the two states and a changing of the scattering length. Figure taken from [3]
length $a$ depends strongly on the magnetic field $B$. Consequently it is a magnetic Feshbach resonance, and the change in scattering length around the resonance can be described by

$$
\begin{equation*}
a(B)=a_{b g}\left(1-\frac{\Delta B}{B-B_{0}}\right) \tag{2.21}
\end{equation*}
$$

with $a_{b g}$ the background scattering length far away from the resonance, $\Delta B$ the resonance width and $B_{0}$ the resonance position. Experimental details on the use of a Feshbach resonance will be given in section 3.2.

### 2.3. Dipolar Interactions in Bose-Einstein Condensates

Chromium has a magnetic dipole moment of $\mu_{m}=6 \mu_{B}$ (with $\mu_{B}$ the Bohr magneton) due to six unpaired valence electrons. As the dipolar interactions scales like $\mu_{m}^{2}$, the dipolar strength for chromium atoms is 36 times stronger than for alkali atoms, which have only one unpaired electron. For chromium therefore, the dipole-dipole interaction becomes relevant for the properties of the BEC, in addition to the contact interaction. In particular, new phenomena peculiar to dipolar ultracold gases appear.

In this section, we first discuss the binary and mean-field description of the dipole-dipole interaction. We then describe the expansion and the stability of a dipolar BEC.


Figure 2.3.: Dipole-dipole interaction. The magnetic dipoles are polarized by an external magnetic field $B$. The interaction is proportional to the distance as $1 / r^{3}$ and dependent on the relative angle $\vartheta$. The dipole-dipole interaction is attractive in a head-to-tail configuration $\left(\vartheta=0^{\circ}\right)$ and repulsive when sitting side-by-side $\left(\vartheta=90^{\circ}\right.$.)

### 2.3.1. Binary Dipole-Dipole Interaction

The interaction potential for two magnetic dipoles polarized in the same direction is given by

$$
\begin{equation*}
U_{d d}(\boldsymbol{r})=\frac{\mu_{0} \mu_{m}^{2}}{4 \pi} \frac{1-3 \cos ^{2} \vartheta}{r^{3}} \tag{2.22}
\end{equation*}
$$

where $\mu_{0}$ is the vacuum permeability and $\vartheta$ is the angle between the polarization direction and the relative position $r$ between the dipoles, see figure 2.3. As seen from equation (2.22) the potential is anisotropic due to the dependence on the relative angle $\vartheta$. The anisotropy can be easily seen if we take two limiting cases. If the dipoles are sitting in a head-totail configuration $\left(\vartheta=0^{\circ}\right)$, they attract each other and therefore lower their potential energy. In contrast if the dipoles are side-by-side $\left(\vartheta=90^{\circ}\right)$, they repel each other and the potential energy is increased. In addition the potential 2.22 decays in $1 / r^{3}$. As a potential $1 / r^{m}$ cannot be replaced by contact-like pseudo potentials when $m \leq 3$ [42], the dipolar interaction is long-range.

To compare contact (section 2.2) and dipolar interactions, we introduce a characteristic dipolar length

$$
\begin{equation*}
a_{d d} \equiv \frac{\mu_{0} \mu_{m}^{2} m}{12 \pi \hbar^{2}}, \tag{2.23}
\end{equation*}
$$

where the constants in $a_{d d}$ are defined such that a homogeneous dipolar condensate becomes unstable to local density perturbations if $a \leq a_{d d}$, as it will be shown in section 2.3.4. Similar to the definition of the contact coupling strength $g$ (equation (2.12)) we can define a dipolar coupling strength $g_{d d}$

$$
\begin{equation*}
g_{d d} \equiv \frac{4 \pi \hbar^{2}}{m} a_{d d}=\frac{\mu_{0} \mu_{m}^{2}}{3} . \tag{2.24}
\end{equation*}
$$

These definitions for the characteristic dipolar length $a_{d d}$ and the dipolar coupling strength


Figure 2.4.: Dipolar mean-field potential for a spherical trap in two dimensions. The dipoles (shown as little magnets) are aligned by an external magnetic field in $z$-direction. The potential is saddle shaped in Thomas-Fermi approximation as given by equation (2.28). For the shown orientation of the polarization axis the BEC will elongate in $z$-direction. Figure taken from [44].
$g_{d d}$ are used to define the relative strength between dipolar and contact interaction

$$
\begin{equation*}
\varepsilon_{d d} \equiv \frac{g_{d d}}{g}=\frac{a_{d d}}{a}=\frac{\mu_{0} \mu_{m}^{2} m}{12 \pi \hbar^{2} a} . \tag{2.25}
\end{equation*}
$$

For chromium the value of the characteristic dipolar length is $a_{d d}=15.2 a_{0}$. The relative strength is $\varepsilon_{d d} \approx 0.16$ [43] at background scattering length $a_{b g}=102.5 a_{0}$ [21]. For comparison, alkali atoms like rubidium ( $a_{b g} \approx 100 a_{0}$ ) with a magnetic dipole moment of $\mu_{m}=1 \mu_{B}$ have a relative strength of the dipolar interaction of $\varepsilon_{d d, R b} \approx 0.007$ [8, chapter 3]. As a reason of the different masses of rubidium and chromium the relative strength is not 36 times larger for chromium. Instead, the relative dipolar strength is $\varepsilon_{d d, C r} / \varepsilon_{d d, R b} \approx 23$ larger.

### 2.3.2. Mean-Field Approximation of Dipolar Interactions

In the previous section, we only took into account the interaction between two dipoles. This section focuses on the many-body description of a dipolar BEC. Again this manybody model is reduced to a mean field approximation and an approach comparable to the one described in section 2.2.2 yields the (non-linear) non-local GPE

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \Psi(\boldsymbol{r}, t)=\left(-\frac{\hbar^{2} \nabla^{2}}{2 m}+V_{e x t}(\boldsymbol{r})+g|\Psi(\boldsymbol{r}, t)|^{2}+\Phi_{d d}(\boldsymbol{r})\right) \Psi(\boldsymbol{r}, t) . \tag{2.26}
\end{equation*}
$$

with the non-local term $\Phi_{d d}$ describing the dipolar contribution to the mean-field potential

$$
\begin{equation*}
\Phi_{d d}(\boldsymbol{r})=\int \mathrm{d}^{3} \boldsymbol{r}^{\prime}\left|\Psi\left(\boldsymbol{r}^{\prime}, t\right)\right|^{2} U_{d d}\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right) \tag{2.27}
\end{equation*}
$$

For example in a spherical harmonic trap ( $\bar{\omega}=\omega_{x}=\omega_{y}=\omega_{z}$ ) and for all dipoles polarized in $z$-direction with an external magnetic field, the dipolar mean field potential in the Thomas-Fermi approximation is given by [45, chapter D] [8, chapter 5.2]

$$
\Phi_{d d}(\boldsymbol{r})=\frac{\varepsilon_{d d} m \bar{\omega}^{2}}{5}\left(1-3 \cos ^{2} \vartheta\right) \cdot \begin{cases}r^{2} & \text { for } r \leq \bar{R}  \tag{2.28}\\ \frac{R^{5}}{r^{3}} & \text { for } r>\bar{R}\end{cases}
$$

with the Thomas-Fermi radii $\bar{R}=R_{x}=R_{y}=R_{z}$. The mean-field potential shows the same anisotropy as the 2-body dipolar potential and has the shape of a saddle, as illustrated in figure 2.4. As a result of equation (2.28) the dipolar BEC density is not spherically symmetric anymore, but elongated in polarization direction of the dipoles to lower the system's energy.

However, it has been shown that the inverted parabolic shape remains [46, 47]. It is remarkable that the inverted parabolic shape still remains even for an asymmetric trap with dipole-dipole interaction.

### 2.3.3. Expansion Dynamics of a Dipolar Bose-Einstein Condensate

As shown in section 2.2.3, the size of a BEC is on the order of few $\mu \mathrm{m}$. For standard imaging systems the resolution is on the order of around $5 \mu \mathrm{~m}$. Therefore, the BEC size is smaller than the optical resolution and it is consequently necessary to expand the BEC before imaging it. This is possible by turning off the external trapping potential: the BEC then falls down under the effect of gravity. During this 'time-of-flight' (TOF), the BEC expands due to the finite momentum distribution in the trap. In the absence of interactions the wave function of the expanded cloud is given by the Fourier transformation of the in-trap wave function. However, the interactions are usually non-negligible and have an impact on the expansion. This influence can even be used to calibrate the scattering length (see section 3.2.1).

Let us first consider the simplest case of the expansion of a non-interacting BEC. As shown in equation (2.7) a non-interacting BEC has a Gaussian wave function. In momentum space, after performing a Fourier transformation, it remains a Gaussian function with momentum width given by $p_{i}=\hbar / l_{i}=\sqrt{m \hbar \omega_{i}}$. This momentum is compatible with the Heisenberg uncertainty principle and accordingly, a narrow spatial distribution implies a higher momentum distribution.

However, in presence of contact interaction, the density distribution is not a Gaussian function anymore. Instead, in the case of a harmonic trap and within Thomas-Fermi approximation the distribution has the shape of an inverted parabola (see equation (2.18)). To get insight on the expansion dynamics of a BEC with contact interaction, we introduce a classical picture. In this case, one can write down the force that the BEC undergoes [48]

$$
\begin{equation*}
\vec{F}(\boldsymbol{r}, t)=-\nabla\left(V_{\text {ext }}(\boldsymbol{r}, t)+g n_{T F}(\boldsymbol{r}, t)\right) . \tag{2.29}
\end{equation*}
$$



Figure 2.5.: Time evolution of a contact interacting condensate (a) The Thomas-Fermi radii evolution for an abrupt switch off of the trap $\left(\omega_{r}, \omega_{z}\right)=2 \pi \cdot(650,270) \mathrm{Hz}$ during a time of flight. A crossover is visible at 0.9 ms (b) Evolution of the condensate aspect ratio $\kappa$. The condensate aspect ratio is initially smaller than unity. During the TOF, it crosses unity at 0.9 ms and becomes $\kappa>1$ for long enough expansion times.

At $t=0$ the BEC is trapped, the overall force is zero and consequently $V_{\text {ext }}(\boldsymbol{r}, 0)+$ $g n_{T F}(\boldsymbol{r}, 0)$ is a constant and equal to $\mu$ as in equation (2.16). For $t>0$ the BEC experiences a dilatation and its time-dependent radii can be written as

$$
\begin{equation*}
R_{i}(t)=R_{i}(0) b_{i}(t) \quad i=(x, y, z) \tag{2.30}
\end{equation*}
$$

where $R_{i}(0)$ are the in-trap values of the Thomas-Fermi radii (given in equation (2.19) and $b_{i}$ the dilatation coefficients. Within the Thomas-Fermi approximation the parabolic shape is conserved during the expansion because of the dominant contact interaction. Writing down Newton's law one gets [48]

$$
\begin{equation*}
\ddot{b}_{i}+\omega_{i}^{2}(t) b_{i}-\frac{\omega_{i}^{2}(0)}{b_{i} b_{x} b_{y} b_{z}}=0 \quad i=(x, y, z) . \tag{2.31}
\end{equation*}
$$

The second term in (2.31) comes from the confining potential with the possibility for time dependent trap frequencies $\omega_{i}(t)$ whereas the third term due to the atomic interaction in the initial trap $\omega_{i}(0)$. We are considering an abrupt turning-off of the initial trap, but these equations 2.31 remain valid for time-dependent traps.

For simplification a cylindrically symmetric trap $V_{\text {ext }}=m / 2\left(\omega_{r}^{2} r^{2}+\omega_{z}^{2} z^{2}\right)$ is used for following considerations with a trap aspect ratio $\lambda=\omega_{z} / \omega_{r}$. After sudden switch-off of


Figure 2.6.: Condensate aspect ratios $A_{1}$ and $A_{2}$ against time of flight for different relative dipolar strengths $\varepsilon_{d d}$. The error bar in $\mathbf{d}$ gives the typical uncertainty (a) 'contact interacting case' with polarization axis in imaging view (b)-(d) For the red (blue) points the polarization axis is aligned in axial (radial) direction (see inset in b). (b) Expansion at background scattering length with $\varepsilon_{d d}=0.16$ similar to [45, 49] (c) $\varepsilon_{d d}=0.5$ (d) $\varepsilon_{d d}=0.75$ with $A_{2}$ staying smaller than unity. The solid lines are theoretical calculations [45]. Figure taken from [10].
the trap $\left(\omega_{i}(t)=0, t>0\right)$, the equations (2.31) simplify to

$$
\begin{equation*}
\ddot{b}_{r}=\frac{\omega_{r}^{2}}{b_{r}^{3} b_{z}}, \quad \ddot{b}_{z}=\frac{\omega_{z}^{2}}{b_{r}^{2} b_{z}^{2}} . \tag{2.32}
\end{equation*}
$$

From the solution of these coupled equations and the initial radii of the BEC, we determine the radii $R_{r}$ and $R_{z}$ of the BEC during the TOF. The evolution of $R_{r}$ and $R_{z}$ for an initial trap with trapping frequencies ${ }^{4}\left(\omega_{r}, \omega_{z}\right)=2 \pi \cdot(650,270) \mathrm{Hz}$ is shown in figure 2.5a. An important quantity for describing the expansion is the condensate aspect ratio $\kappa=R_{r} / R_{z}$. For a contact interacting BEC in a trap, the condensate aspect ratio is equal to the trap aspect ratio: $\kappa=\lambda=\omega_{z} / \omega_{r}$, as mentioned in equation (2.19). When the condensate aspect ratio is initially (in-trap) $\kappa<1(\kappa>1)$, it becomes $\kappa>1(\kappa<1)$ for sufficiently long expansion times (see figure 2.5 b ).

Finally, the expansion of a dipolar BEC is much more complicated, even though the BEC shape remains parabolic within the Thomas-Fermi approximation. As learned in

[^2]section 2.3.2, the dipolar BEC is elongated in the polarization direction. For a dipolar BEC in its trap, the relation between $\kappa$ and $\lambda$ is not as simple as in the case of a purely contact interacting BEC and depends on the relative orientation of the trap axis and the polarization direction of the dipoles. As a consequence, the expansion of the dipolar BEC depends also on the orientation of the trap compared to the polarization direction of the dipoles, as can be seen in figure 2.6b-d. For an initially prolate BEC, the expanded condensate aspect ratio was observed to be smaller (larger) for the polarization axis aligned along the axial (radial) direction [45, 49]. This effect is enhanced if the relative strength of the dipolar interaction $\varepsilon_{d d}$ is increased. Finally for a relative strength $\varepsilon_{d d}=$ 0.75 and a prolate trap $(\lambda<1)$ with the dipoles polarized along the trap axis, it was possible to keep the condensate aspect ratio below unity during the TOF. Note that if the polarization axis of the dipoles is aligned along the radial direction of the trap and along the imaging axis, the observed condensate aspect ratio behaves like the one of a purely contact interacting BEC (see figure 2.6a).

### 2.3.4. Stability of a Dipolar Bose-Einstein Condensate

As discussed in section 2.2.4, it is possible to tune the scattering length $a$ with a Feshbach resonance. For a purely contact interacting BEC, the stability strongly depends on the scattering length. As the dipolar interaction is anisotropic, the stability of a dipolar BEC depends not only on the value of the scattering length $a$ but also on the geometry of the external trapping potential. One finds the stable regions theoretically by looking for positive local minima in the non-local Gross-Pitaevskii energy functional

$$
\begin{equation*}
E(\Psi)=\int\left[\frac{\hbar^{2}}{2 m}|\nabla \Psi|^{2}+V_{e x t}|\Psi|^{2}+\frac{g}{2}|\Psi|^{4}+\frac{1}{2}|\Psi|^{2} \int U_{d d}\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)\left|\Psi\left(\boldsymbol{r}^{\prime}\right)\right|^{2} \mathrm{~d} \boldsymbol{r}^{\prime}\right] \mathrm{d} \boldsymbol{r} . \tag{2.33}
\end{equation*}
$$

The first energy term corresponds to the quantum pressure, the second to the trapping potential, the third to the contact interaction and the last term is the dipolar interaction energy. To find the minima one can solve the energy functional numerically but there is also an analytical solution using a Gaussian ansatz for the wave function. Similar to equation (2.7) a Gaussian wave function is considered and for simplicity in a cylindrically symmetric trapping potential $V_{\text {ext }}=m / 2\left(\omega_{r}^{2} r^{2}+\omega_{z}^{2} z^{2}\right)$ with trap aspect ratio $\lambda=\omega_{z} / \omega_{r}$

$$
\begin{equation*}
\Psi(r, z)=\left(\frac{N}{\pi^{3 / 2} \sigma_{r}^{2} \sigma_{z} a_{h o}^{3}}\right)^{1 / 2} e^{-\frac{1}{2 a_{h o}^{2}}\left(\frac{r^{2}}{\sigma_{r}^{2}}+\frac{z^{2}}{\sigma_{z}^{2}}\right)} \tag{2.34}
\end{equation*}
$$

with $\sigma_{r}$ and $\sigma_{z}$ the dimensionless variational parameters. With this Gaussian ansatz one can calculate the different energies in the system. The quantum pressure term writes [Methods in 11]

$$
\begin{equation*}
\frac{E_{\mathrm{qp}}}{\hbar \bar{\omega}}=\frac{N}{4}\left(\frac{2}{\sigma_{r}^{2}}+\frac{1}{\sigma_{z}^{2}}\right), \tag{2.35}
\end{equation*}
$$

the potential energy

$$
\begin{equation*}
\frac{E_{\mathrm{pot}}}{\hbar \bar{\omega}}=\frac{N}{4 \lambda^{2 / 3}}\left(2 \sigma_{r}^{2}+\lambda^{2} \sigma_{z}^{2}\right) \tag{2.36}
\end{equation*}
$$

and the mean-field interaction energy (contact and dipolar interaction)

$$
\begin{equation*}
\frac{E_{\text {contact }}+E_{\mathrm{DI}}}{\hbar \bar{\omega}}=\frac{N^{2} a_{d d}}{\sqrt{2 \pi} a_{h o}} \frac{1}{\sigma_{r}^{2} \sigma_{z}}\left(\frac{a}{a_{d d}}-f(\kappa)\right) \tag{2.37}
\end{equation*}
$$

with

$$
\begin{equation*}
f(\kappa)=\frac{1+2 \kappa^{2}}{1-\kappa^{2}}-\frac{3 \kappa^{2} \operatorname{arctanh} \sqrt{1-\kappa^{2}}}{\left(1-\kappa^{2}\right)^{3 / 2}} . \tag{2.38}
\end{equation*}
$$

and the condensate aspect ratio $\kappa=\sigma_{z} / \sigma_{r}$. The function $f(\kappa)$ [45] decreases monotonically, has the asymptotic values $f(0)=1$ and $f(\infty)=-2$ and crosses the zero at $\kappa=1$.

For decreasing values of the scattering length $a$, we check if the energy functional has a positive global or local minimum. The presence of a minimum corresponds to the presence of a stable ground state. If there is no minimum, the BEC is unstable against small density perturbations and will collapse. The scattering length below which the BEC is always unstable, is defined as the critical scattering length $a_{\text {crit }}$.

Let us first describe the case of a purely contact interacting BEC. In this case $f(\kappa)$ is equal to zero and the variational parameters $\sigma_{r}$ and $\sigma_{z}$ are fixed through $\lambda=\omega_{z} / \omega_{r}=$ $\sigma_{r}^{2} / \sigma_{z}^{2}=\kappa^{2}$ (see equation (2.8)). Note that this relation is different in the Gaussian ansatz and in the Thomas-Fermi regime (see equation 2.19). The exact value of $a_{\text {crit }}$ can be determined by searching for a positive global or local minimum, however one can get already the critical scattering length with a good approximation by searching for a change of sign of the total energy. Here, the total energy can be below zero only for negative scattering length $a$. Because $\int|\Psi(\boldsymbol{r})|^{2} d \boldsymbol{r}=N$, the contact interaction energy is proportional to $N^{2}$ and will dominate the energy term for high number of atoms, as the positive energy contribution from the external potential and quantum pressure depend only linearly on $N$. Hence, a contact interacting BEC with many atoms is unstable at negative scattering length and the critical scattering length $a_{\text {crit }}$ is close to zero. However, for a small number of atoms, i.e. very dilute gas, the interactions do not play such a dominant role and the BEC can be stabilized by the external trap and the quantum pressure. Therefore, it is possible to realize stable BECs at negative scattering lengths. This is indeed what was done to realize a lithium BEC at background scattering length of $a_{b g} \approx-27 a_{0}[50]$. The number of atoms in the BEC was around 650-1300 atoms leading to very low densities.

Let us now consider the case of a dipolar BEC. For polarized dipoles in $z$-direction the dipolar interaction energy is negative in a prolate trap $(\lambda<1$, dipoles mainly sitting head-to-tail) and positive in a oblate trap ( $\lambda \gg 1$, dipoles are side-by-side). In the interaction dominated regime $\left(N a_{d d} \gg a_{h o}\right)$ the asymptotic critical scattering lengths for very prolate or oblate traps are given by the term in brackets in equation 2.37. For a



Figure 2.7.: (a) Experimental (green squares) and theoretical (green line) values for the critical scattering length against the trap aspect ratio $\lambda=\omega_{z} / \omega_{r}$ for a chromium BEC with 20,000 atoms. The grey line is the asymptotic stability boundary for many atoms ( $N a_{d d} \gg a_{h o}$ ) with the limt values $a_{d d}$ and $2 a_{d d}$. The pure contact interacting case is plotted in red with otherwise chromium parameters. (b)-(e) Behaviour of the energy landscape. Lines of equal energy for a constant trap aspect ratio $\lambda=10$ for different scattering length (marked in figure a as blue dots). (b) For $a>a_{d d}$ (figure b) there is a global minimum. (c,d) For $a_{\text {crit }} \leq a<a_{d d}$ the collapsed ground state ( $\sigma_{r} \rightarrow 0$ at finite $\sigma_{z}$ ) emerges and the BEC is metastable.
(e) For $a<a_{\text {crit }}$ there is no local minima left. Figure taken from [11].
very prolate trap, $f(0)=1$ and the critical scattering length tends to $a_{\text {crit }}=a_{d d}$, which therefore justifies the definition of $a_{d d}(2.23)$. For a very oblate trap the asymptotic critical scattering length is $a_{\text {crit }}=-2 a_{d d}$. This means that the dipolar interaction destabilizes the BEC in a prolate trap and stabilizes it in an oblate trap. Therefore, in principle it is possible to have stable BECs with high atom numbers and negative scattering length, due to the stabilizing dipolar interaction. For decreasing scattering length the relative dipole strength is enhanced and the density distribution is modified by the dipolar meanfield interaction (section 2.3.3). Near the critical scattering length $a_{\text {crit }}$ the BEC shrinks strongly in radial direction until the instability point is reached. This effect is also visible in the energy landscape of the BEC with a Gaussian ansatz (see figure 2.7b-e). For scattering lengths $a>a_{d d}$ a global minimum is visible, whereas for $a<a_{d d}$, a collapsed ground state emerges with $\sigma_{r} \rightarrow 0$ at finite $\sigma_{z}$.

This was also experimentally investigated in [11] in a cylindrically symmetric trap with mean frequency $\bar{\omega}=2 \pi \cdot 700 \mathrm{~Hz}$. The stability diagram for a chromium BEC with 20,000 atoms was mapped for trap aspect ratios $\lambda=0.1-10$ (see figure 2.7). The critical scattering length $a_{\text {crit }}$ was identified through the abruptly vanishing BEC atom number at the critical scattering length $a_{\text {crit }}$. A purely dipolar BEC was obtained in an oblate trap $\lambda=10$ that corresponds to a condensate aspect ratio of $\kappa=1$. However, the regime

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with negative scattering length $a<0$ could not be reached.

## 3. Experimental Setup

In this section we briefly describe the experimental setup ${ }^{1}$ and the procedures needed to generate a ${ }^{52} \mathrm{Cr}$ BEC. The experimental setup has been extensively discussed in [51, 52], PhD theses [53-58] or diploma theses [59-61]. In section 3.1 the production of a chromium Bose-Einstein condensate is explained, followed by experimental details on the use of the Feshbach resonance and the procedure for calibrating the scattering length.

### 3.1. Creating a Chromium Bose-Einstein Condensate

We give an overview of the steps used to generate a ${ }^{52} \mathrm{Cr}$ BEC. In the next sections each step is discussed in more detail. The first ${ }^{52} \mathrm{Cr}$ BEC was achieved in 2005 [7, 62] in Stuttgart with the following cooling and trapping steps:

1. deceleration of hot atoms $\left(T=1475^{\circ} \mathrm{C}\right)$ with a Zeeman slower (ZS)
2. cooling in a modified magneto-optical trap (MOT) and continuous loading in a magnetic trap (MT)
3. Doppler cooling and radio frequency ( RF ) induced evaporative cooling in the magnetic trap
4. transfer of the atoms into a crossed optical dipole trap (ODT) and optical pumping to the magnetic substate with the lowest energy
5. forced evaporative cooling in the ODT towards the critical temperature

With these steps I will shortly describe the different parts of the experimental setup (see figure 3.1) in this chapter.

### 3.1.1. Cooling Chromium in a Magnetic Trap

The experiment consists of two vacuum chambers: the oven chamber and the main chamber which are connected by a Zeeman slower (ZS), see figure 3.1. The chromium atoms are sublimated from a high-purity sample of chromium (99.99\%) by a high temperature effusion cell at $T=1475^{\circ} \mathrm{C}$ in the oven chamber, leading to a chromium beam with a

[^3]

Figure 3.1.: Schematic scheme of used setup. (a) The whole apparatus is shown: the oven chamber connected via the Zeeman slower to the main chamber. The modified magneto-optical trap consists of the cloverleaf coils (that produce also the magnetic trap) and six cooling beams. The probe beam for the absorption imaging is in $x$-direction. (b) The scheme is rotated by $90^{\circ}$. The two optical dipole traps are orientated in $z$ - and $y$-direction. The optical pumping beam propagates also in $y$-direction. Figure taken from [62]


Figure 3.2.: Level scheme of ${ }^{52} \mathrm{Cr}$ and experimental sequence. (a) Zeeman slower and MOT use the transition ${ }^{7} \mathrm{~S}_{3} \leftrightarrow{ }^{7} \mathrm{P}_{4}(425.6 \mathrm{~nm})$. The atoms from the excited state decay with a ratio of $1 / 250,000$ to the metastable state ${ }^{5} \mathrm{D}_{4}$. Consequently, this metastable state is continuously loaded in the magnetic trap. After the MOT-light is switched off the atoms in the magnetic trap are repumped to the state ${ }^{7} \mathrm{P}_{3}$ with a repumper at 663.2 nm . (b) To prevent dipolar relaxations the atoms are transferred into an ODT and optically pumped to the lowest magnetic substate $m_{J}=-3$ with $\sigma^{-}$ light via the transition ${ }^{7} \mathrm{~S}_{3} \leftrightarrow{ }^{7} \mathrm{P}_{3}(427.6 \mathrm{~nm})$.
mean Boltzman velocity of $\bar{v} \approx 900 \mathrm{~m} / \mathrm{s}$. The spin-flip ZS [60, chapter 3.4] decelerates all atoms with a velocity below $v_{\max } \approx 580 \mathrm{~m} / \mathrm{s}$ via resonant laser cooling. At the end of the ZS , the velocity of these atoms is decreased down to the MOT capture velocity of $\approx$ $30 \mathrm{~m} / \mathrm{s}$. Furthermore the ZS acts as a differential pumping stage to have a two orders of magnitude lower pressure ( $\approx 10^{-11} \mathrm{mbar}$ ) in the main chamber than in the oven chamber ( $\approx 10^{-9} \mathrm{mbar}$ ).

Because the ZS only acts on the longitudinal velocity, a two-dimensional optical molasses in radial directions is used at the entrance of the ZS [63, chapter A.1]. This cools the chromium atoms in the radial directions and collimates the atomic beam. Hence, the atomic flux is increased and finally the atom number in the MOT is higher.

The decelerated chromium atoms are captured in a modified MOT [51, 64] where the magnetic field is created by a Ioffe-Pritchard magnetic trap with cloverleaf shaped coils. A magnetic trap makes use of the interaction between the magnetic moment of the atoms and the static magnetic field. The used Ioffe-Pritchard trap has a parabolic shaped magnetic potential, at least around its minimum. The MOT is operating at the transition ${ }^{7} \mathrm{~S}_{3} \leftrightarrow$ ${ }^{7} \mathrm{P}_{4}(425.6 \mathrm{~nm})$ with a density-limited atom number of around $5 \cdot 10^{6}$. This limit is due to the fact that chromium suffers a large inelastic two-body loss coefficient for excited state collisions. The low atom numbers in the MOT are not sufficient for reaching Bose-Einstein condensation in chromium.

Therefore a standard MOT mechanism is not applicable. However, chromium decays from the excited state ${ }^{7} \mathrm{P}_{4}$ into a metastable state ${ }^{5} \mathrm{D}_{4}$ with a branching ratio of $1 / 250,000$ compared to the decay into the ground state ${ }^{7} \mathrm{~S}_{3}$. The metastable state ${ }^{5} \mathrm{D}_{4}$ is decoupled
from the MOT light (see figure 3.2a). Most of the atoms in state ${ }^{5} \mathrm{D}_{4}$ are already cooled down to the Doppler temperature of $124 \mu \mathrm{~K}$ [57] and therefore the low-field seeking magnetic substates $\left(m_{J}>0\right)$ are trapped in the MT. To trap more metastable atoms, the MT volume is increased by using a two-dimensional MOT plus a one-dimensional optical molasses instead of a conventional three-dimensional MOT. During the MOT phase, around $2 \cdot 10^{8}$ atoms accumulate in the metastable ${ }^{5} \mathrm{D}_{4}$ state. After 6 s we switch the MOT light off.

Then we repump the metastable atoms to the ground state ${ }^{7} \mathrm{~S}_{3}$ state via the state ${ }^{7} \mathrm{P}_{3}$ with a 663.2 nm laser. Subsequently the MT field is increased adiabatically in 1 s to compress the atom cloud. This heats up the sample to around 1 mK . This compressed cloud is then Doppler cooled to a temperature of $T \approx 170 \mu \mathrm{~K}$ by applying a short resonant laser pulse. The MT field is again decreased to prepare the radio frequency (RF) induced evaporative cooling [65]. An additional magnetic offset field is needed to prevent Majorana spin-flip transitions to non-trappable magnetic substates ( $m_{J} \leq 0$ ). The role of RF cooling is to remove selectively the hottest atoms and the remaining atoms re-thermalize to a lower temperature and higher spatial densities.

For commonly used alkali atoms, the RF cooling can be performed until degeneracy is reached. But for chromium atoms, this is prevented by the dipolar relaxation [66] process which increases for higher spatial densities. Because dipolar collisions do not conserve angular momentum the atoms relax to $m_{J} \leq 0$. This process heats up the sample and induces losses as states with $m_{J} \leq 0$ are not magnetically trappable. For this reason, the chromium atoms are only precooled in 8.5 s to $40 \mu \mathrm{~K}$ where we still have around $10^{7}$ atoms in the trap.

To reach quantum degeneracy all atoms have to be in the energetic lowest magnetic substate $m_{J}=-3$ where spin changing collisions are energetically suppressed. Therefore, after the RF cooling process, the atoms are transferred into a horizontal optical dipole trap (ODT1) along the $z$-direction. ODTs make use of the interaction between the induced electric dipole moment of the atom and an oscillating electric field (AC Stark effect, see section 4.1), and are usually created with a focused Gaussian laser beam. This trapping mechanism is independent of the particular magnetic substate and it is possible to trap atoms in the $m_{J}=-3$ substate. The transfer efficiency from the MT to the ODT1 is $40 \%$. The atoms are then optically pumped with a $\sigma^{-}$-polarized light ( 427.8 nm ) to the magnetic substate with the lowest energy (see figure 3.2 b ). A second vertical optical dipole trap (ODT2) in $y$-direction is then ramped up to create a crossed ODT.

### 3.1.2. Condensation of Chromium in a crossed Optical Dipole Trap

After loading into the horizontal ODT1 and ramping up the vertical ODT2 roughly 600,000 atoms are trapped inside the crossed region at a temperature of $15 \mu \mathrm{~K}$. To reach quantum degeneracy, forced evaporation is used by decreasing the ODT intensities, which are controlled with acousto-optic modulators (AOMs). As a part of this thesis the twofrequency AOM drivers $[34,61]$ were renewed, improving the pointing stability of the two

ODT beams (see appendix A.1).
When we want to use the Feshbach resonance (section 3.2), we ramp up the magnetic field to around 600 G along the $z$-direction. Along the way we pass 14 Feshbach resonances. Near each resonance occur inelastic 3-body losses, which depend crucially on the density. To prevent high atom losses we have to pass all the 14 resonances fast enough with a low spatial density. This is the case during the forced evaporation phase in the crossed ODT before reaching degeneracy and we thus ramp up the magnetic field during this phase.

The forced evaporation is finished when ODT1 and ODT2 are both decreased to trapping frequencies of $\left(\omega_{x}, \omega_{y}, \omega_{z}\right)=2 \pi \cdot(285,210,195) \mathrm{Hz}$. The forced evaporation takes 5.8 s and the total experimental sequence takes 23 s . We reach a BEC containing typically 20,000 condensed atoms. After generating the BEC we can modify the trapping potential with the crossed ODT and change the Feshbach magnetic field to adjust the scattering length $a$.

The density distribution after a time of flight (TOF) is recorded by absorption imaging. A resonant probe laser beam is shone on the atoms in $x$-direction. Due to atom-photon scattering, the probe beam is absorbed by the atoms and the 'shadow' of the atomic cloud is recorded on a CCD-Camera. From this image we calculate the optical density of the atomic cloud, which is integrated along the $x$-direction. Depending if we are in low or high magnetic field (for the Feshbach resonance) the probe beam polarization is different:

The low magnetic field is oriented along the imaging axis so that the probe beam polarization is well-defined to drive single atomic transitions. We use circularly polarized $\sigma^{-}$-light resonant to the transition $\left.\left.\left.\right|^{7} \mathrm{~S}_{3}, m_{J}=-3\right\rangle\left.\rightarrow\right|^{7} \mathrm{~S}_{3}, m_{J}=-4\right\rangle$ for imaging.

If the high magnetic field for the Feshbach resonance is turned on during the imaging, the magnetic field is perpendicular to the imaging axis. Therefore, the probe beam polarization is not well-defined along the imaging axis for the atoms. To avoid this problem we use linearly polarized $\pi$-light (polarized in $x$-direction), which is a superposition of $\sigma^{-}$and $\sigma^{+}$for the atoms. The $\sigma^{-}$light drives the same transition as mentioned before, whereas the $\sigma^{+}$light is far detuned from the transition $\left.\left.\left|{ }^{7} \mathrm{~S}_{3}, m_{J}=-3\right\rangle \rightarrow\right|^{7} \mathrm{~S}_{3}, m_{J}=-2\right\rangle$ and its absorption is consequently very weak. Hence, only half of the probe beam intensity can be absorbed at maximum. This has to be taken into account for calculating the optical density [61].

### 3.2. Enhance the Effect of Dipoles via a Feshbach Resonance

As mentioned in section 2.3, the dipolar strength is only a perturbative effect at background scattering length. However, the contact interaction strength can be decreased by a Feshbach resonance (see section 2.2.4) leading to an enhancement of dipolar effects. Near Feshbach resonances, inelastic 3-body losses are enhanced. By monitoring inelastic losses in a system of ultracold chromium atoms, 14 Feshbach resonances were found for magnetic
fields below 600 G [67]. The 14th one is the broadest, is centered at $B_{0}=589.1 \mathrm{G}$ and has a width of $\Delta B=1.6 \mathrm{G}$ [33] (measured value, the theoretical value is $\Delta B=1.7 \mathrm{G}$ ). We use this particular resonance as it allows the best control on the scattering length.

To produce a homogeneous magnetic field, Helmholtz coils and currents in the order of $I=400 \mathrm{~A}$ are used. To compensate the curvature of the field created by the Helmholtz coils, we superimpose a second magnetic field. For this we use the pair of coils that is used for the MT in the earlier stage of the experiment. To create a stable and a mostly noiseless magnetic field at 590 G , the current is actively stabilized. For this, the resistance of a MOSFET (metal-oxide-semiconductor field-effect transistor) is controlled via an electronic stabilization loop. The current is stabilized to a peak-to-peak noise of $\Delta I / I<5 \cdot 10^{-5}$, while the RMS noise is $\Delta I_{R M S} / I<1 \cdot 10^{-5}$ [58, chapter 4.2].

### 3.2.1. Calibration of the Scattering Length

In our experiment the scattering length is calibrated using the BEC expansion (section 2.3.3). As shown in equations (2.19) and (2.20) for a purely contact interacting BEC the initial radii $R_{i}(t=0) \propto(N a)^{1 / 5}$ are depending on the scattering length. During the expansion the initial trapping energy, proportional to the chemical potential $\mu \propto(N a)^{2 / 5}$ (see equation (2.29)), is completely transformed into kinetic energy. Once this is done, the BEC expands linearly and its expansion velocity has the same dependence on the scattering length as the initial radii $v \propto(N a)^{1 / 5}[58$, chapter 4.4.2]. The scattering length can then be determined experimentally by measuring the BEC radii $R_{i}\left(t_{T O F}\right)$ after the expansion time $t_{\text {TOF }}$

$$
\begin{equation*}
a \propto \frac{R_{i}^{5}\left(t_{T O F}\right)}{N} \quad i=(x, y, z), \tag{3.1}
\end{equation*}
$$

In the case of a dipolar BEC the expansion is described analytically in Thomas-Fermi approximation in [45] with some corrections in [41, chapter A.5.9.]. For a dipolar BEC the expansion is still linear against the scattering length and the calculated value $R^{5} / N$ is shown in figure 3.3a. Both the radial and axial value show a linear dependence on the scattering length until $a=30 a_{0}$. The radial value $R_{y}^{5} / N$ is even linearly decreasing until the stability threshold is reached (around $a=a_{d d}$ ) and allows to approximate for all scattering lengths

$$
\begin{equation*}
\frac{R_{y}^{5}}{N}=m \cdot\left(a-a_{\mathrm{offset}}\right), \tag{3.2}
\end{equation*}
$$

where the parameters $m$ and $a_{\text {offset }}$ can be calculated and depend on the trapping frequencies. Therefore, to calibrate the scattering length, we choose to measure the radial radius $R_{y}$ after TOF. We use a prolate trapping potential $\left(\omega_{x}, \omega_{y}, \omega_{z}\right)=2 \pi \cdot(680,624,270) \mathrm{Hz}$ where the radial radius after the expansion is larger than the axial radius due to the stronger confinement (see figure 2.5). We image the BEC at different magnetic fields $B$ after a TOF of 7 ms and extract the atom number of the BEC and its radius $R_{y}$. Additionally for measuring the BEC radius precisely, a good calibration of the imaging system is needed which is explained in section 4.5. The measured values of $R_{y}^{5} / N$ at different
magnetic field $B$ around $B_{0}=589.1 \mathrm{G}$, are mapped onto the scattering lengths using the linear dependence of $R^{5} / N$ on $a$. Due to the uncertainties on the BEC atom number and on the trapping frequencies, we introduce an effective proportionality factor $m_{\exp }$ in equation 3.2. $m_{\text {exp }}$ can be calculated far away from the Feshbach resonance ${ }^{2}$ where the scattering length is close to the background scattering length $a_{b g}=102.5 a_{0}$

$$
\begin{equation*}
m_{\exp }=\left.m \cdot\left[\frac{R_{y}^{5}}{N}\right]\right|_{\text {exp,bg }} /\left.\left[\frac{R_{y}^{5}}{N}\right]\right|_{\text {theo,bg }} \tag{3.3}
\end{equation*}
$$

With this mapping the dependency of the scattering length on the magnetic field is given by

$$
\begin{equation*}
a(B)=\frac{\left[R_{y}^{5} / N\right](B)}{m_{\exp }}+a_{\mathrm{offset}} \tag{3.4}
\end{equation*}
$$

Finally the measured scattering length is fitted to the characteristic shape of a Feshbach resonance, as in equation (2.21) and is shown in figure 3.3b. The uncertainty on the scattering length $a$ is given by the fit errors. For stability measurements (presented in chapter 5) we want to decrease the scattering length to values between $a_{d d}$ to $-2 a_{d d}$ (15 to $-30 a_{0}$ ). For positive scattering lengths below $a_{d d}$ the uncertainty is $\Delta a \approx 1.5 a_{0}$ and for negative scattering lengths between $0 a_{0}>a>-2 a_{d d}$ we obtain $\Delta a \approx 2-3 a_{0}$.

[^4]

Figure 3.3.: Calibration of the scattering length. (a) The value $R^{5} / N$ is calculated within the Thomas-Fermi approximation for a BEC containing 20,000 atoms at different scattering length. Both the radial and the axial value of $R^{5} / N$ depend linearly on the scattering length. (b) The scattering length extracted from the measured values $R_{y}^{5} / N$ is plotted against the magnetic field $B$ around the resonance $B_{0}=589.1 \mathrm{G}$. The error bars correspond to the standard deviation over five measurements. The red line is a fit with the characteristic shape of a Feshbach resonance (equation (2.21)). Both figures are for a prolate trap with trapping frequencies $\left(\omega_{x}, \omega_{y}, \omega_{z}\right)=2 \pi \cdot(680,624,270) \mathrm{Hz}$ and an expansion time of 7 ms . The green circled data points are used to calculate $m_{\text {exp }}$ (see text).

## 4. One-dimensional Optical Lattices

Optical lattices are periodic potentials for ultracold atoms typically created by interfering laser beams. They are extensively used to mimic solid state physics using cold atoms, where the ultracold atoms in an optical lattice play the role of electrons in an atomic lattice $[2,68]$. Optical lattices are also used to confine atoms with high trapping frequencies [1]. In our experiment we use a one-dimensional optical lattice to investigate the properties of a dipolar BEC in such periodic potentials. In contrast to the atomic lattice of a solid body, the depth of an optical potential can be precisely tuned by the intensity of the laser beams creating it. This introduces great possibilities for experimental investigations, however they crucially depend on the calibration of the lattice depth. This requires the knowledge about the fundamental properties of an optical lattice and well suited calibration methods.

In this chapter, we first present some general properties and characteristics of optical lattices. Then we describe in more detail our configuration in section 4.3. Subsequently, in section 4.4, we describe two different, independent methods to calibrate the depth of the optical lattice potential: first via interference patterns and second through the diffraction of a BEC from a pulsed lattice. Besides this, the diffraction patterns were used to precisely measure the scaling of our imaging system as shown in section 4.5.

### 4.1. Optical Dipole Traps

An optical lattice typically involves two interfering laser beams, which individually would provide a standard optical dipole trap (ODT). Therefore, we first give a short overview on the working principle of ODTs before we explain the characteristics of an optical lattice. A detailed description of ODTs is given in the review [69].

A light wave described by an electric field $\boldsymbol{E}$ oscillating at frequency $\omega$ induces an electric dipole moment $\boldsymbol{d}$ in the atom, which is proportional to the dynamic polarizability $\alpha$. This leads to an energy shift of the atomic energy levels via the AC Stark effect and the corresponding dipole potential is then described by

$$
\begin{equation*}
V_{d i p}=-\frac{1}{2}\langle\boldsymbol{d} \boldsymbol{E}\rangle=-\frac{1}{2 \varepsilon_{0} c} \alpha(\omega) I, \tag{4.1}
\end{equation*}
$$

with the field intensity $I=2 \varepsilon_{0} c|E|^{2}$ and $\varepsilon_{0}$ the vacuum permittivity. The factor $1 / 2$ takes into account that the electric dipole is induced and not permanent. The polarizability $\alpha$ can be calculated in a two level scheme considering the Lorentz oscillator model, where the electron is elastically bound to the nucleus featuring an eigenfrequency $\omega_{0}$. The radiative
energy loss of the accelerated electron is taken into account by a damping rate $\Gamma$ that corresponds to the decay rate of the excited state. The corresponding equation of motion is $\ddot{x}+\Gamma \dot{x}+\omega_{0}^{2} x=-e E(t) / m_{e}$ with the driving field $E(t)=E_{0} \cos (\omega t)$. The polarizability is then proportional to the amplitude $A(\omega)$ of this driven harmonic oscillator and is given by $-e A(\omega)$.

For large detunings $\delta=\omega-\omega_{0}$ the scattering rate of photons is much smaller than the spontaneous decay rate $\Gamma$. In this case the dipole potential is given by [69]

$$
\begin{equation*}
V_{d i p}(\boldsymbol{r})=-\frac{3 \pi c^{2}}{2 \omega_{0}^{3}}\left(\frac{\Gamma}{\omega_{0}-\omega}+\frac{\Gamma}{\omega_{0}+\omega}\right) I(\boldsymbol{r}) . \tag{4.2}
\end{equation*}
$$

In the case $|\delta| \ll \omega_{0}$, the rotating-wave approximation is valid. Then the second term in the brackets of equation (4.2) is much smaller than the first term and can therefore be neglected. The dipole potential simplifies to

$$
\begin{equation*}
V_{d i p}(\boldsymbol{r})=\frac{3 \pi c^{2}}{2 \omega_{0}^{3}} \frac{\Gamma}{\delta} I(\boldsymbol{r}), \tag{4.3}
\end{equation*}
$$

hence, for a red detuned light field $(\delta<0)$ the dipole potential is negative and attracts atoms into regions of maximum intensity. The opposite effect is true for the blue detuned case $(\delta>0)$ where potential minima correspond to minima of intensity. Thus atoms can be trapped by a red-detuned focused laser beam. The transversal shape of a laser beam is generally given by a Gaussian function. In the following paragraph we will give a short overview on the propagation of a Gaussian beam and extract the trapping frequencies and other characteristics of real light potentials.

## Gaussian Beams and Trap Geometry

The intensity profile $I(\boldsymbol{r})$ of a cylindrically symmetric Gaussian beam is given by

$$
\begin{equation*}
I(r, z)=I_{0}\left(\frac{w_{0}}{w(z)}\right)^{2} e^{-\frac{2 r^{2}}{w^{2}(z)}} \tag{4.4}
\end{equation*}
$$

with $z$ the direction of propagation, $r$ the radial coordinate and $I_{0}$ the maximum intensity. The width $w(z)$ is defined as the value where the radial intensity of the beam drops down to a value $1 / e^{2}$, when compared to the corresponding axial value. Along the direction of propagation, the width is given by

$$
\begin{equation*}
w(z)=w_{0} \sqrt{1+\left(\frac{z}{z_{R}}\right)^{2}} \tag{4.5}
\end{equation*}
$$

with the beam waist $w_{0}$ in the focus. The characteristic distance

$$
\begin{equation*}
z_{R}=\frac{\pi w_{0}^{2}}{\lambda} \tag{4.6}
\end{equation*}
$$

is called the Rayleigh length and denotes the distance where the width has increased by a factor of $\sqrt{2}$ with respect to the waist $w_{0}$. For given laser power $P$ and beam waist $w_{0}$ the maximum intensity in the focus $(z=0, r=0)$ is

$$
\begin{equation*}
I_{0}=\frac{2 P}{\pi w_{0}^{2}} \tag{4.7}
\end{equation*}
$$

According to equation (4.3) the dipole potential $V_{\text {dip }}$ is directly proportional to the field intensity and therefore

$$
\begin{equation*}
V_{d i p}(r, z)=V_{0}^{d i p}\left(\frac{w_{0}}{w(z)}\right)^{2} e^{-\frac{2 r^{2}}{w^{2}(z)}}, \tag{4.8}
\end{equation*}
$$

with the trap depth $V_{0}^{\text {dip }}$ that can be calculated using $I_{0}$ and equation (4.3). For trap depths much larger than the thermal atom energy $k_{B} T$ the atoms will stay in the center of the trap. In this case the trapping potential is well approximated by a harmonic oscillator potential $V_{\text {dip }}=m / 2\left(\omega_{r}^{2} r^{2}+\omega_{z}^{2} z^{2}\right)$ with the radial and axial trapping frequencies

$$
\begin{equation*}
\omega_{r}=\sqrt{\frac{4 V_{0}^{d i p}}{m w_{0}^{2}}}, \quad \omega_{z}=\sqrt{\frac{2 V_{0}^{d i p}}{m z_{R}^{2}}} . \tag{4.9}
\end{equation*}
$$

Typical trapping frequencies in a single beam trap are a few Hz axially and $100-1000 \mathrm{~Hz}$ radially. From an experimentalists point of view, it is important to note that the trap depth depends linearly on the power in the laser beam, while the trapping frequencies scale like $\sqrt{P}$.

### 4.2. Characteristics of a one-dimensional Optical Lattice

Creating an optical lattice is based on the principles of an ODT, but requires interfering laser beams. To have a standing light wave requires two use beams with wave vectors $\boldsymbol{k}_{L}$ and $\boldsymbol{k}_{L}^{\prime}$, with linear polarization in the same plane ${ }^{1}$ (described by the unity vector $\hat{\boldsymbol{e}}_{y}$ ) and same wavelength $\lambda_{L}\left(\omega=\omega^{\prime}\right)$. The electric field created by the superposition of these two laser beams is

$$
\begin{align*}
\boldsymbol{E}(\boldsymbol{r}, t) & =E_{0} \hat{\boldsymbol{e}}_{y}\left[\cos \left(\omega t+\boldsymbol{k}_{L} \boldsymbol{r}\right)+\cos \left(\omega^{\prime} t+\boldsymbol{k}_{L}^{\prime} \boldsymbol{r}\right)\right] \\
& =2 E_{0} \hat{\boldsymbol{e}}_{y}\left[\cos \left(\frac{2 \omega t+\boldsymbol{r}\left(\boldsymbol{k}_{L}-\boldsymbol{k}_{L}^{\prime}\right)}{2}\right) \cdot \cos \left(\frac{\boldsymbol{r}\left(\boldsymbol{k}_{L}+\boldsymbol{k}_{L}^{\prime}\right)}{2}\right)\right] . \tag{4.10}
\end{align*}
$$

The laser beams are crossing under an angle $\theta$, where $\theta$ is defined as shown in figure 4.1. We define the direction $\hat{\boldsymbol{e}}_{z}$ as the angle bisector of $\theta$. The optical lattice field in $z$-direction is in this case

$$
\begin{equation*}
\boldsymbol{E}(z, t)=2 E_{0} \hat{\boldsymbol{e}}_{y}\left[\cos (\omega t) \cos \left(z k_{L} \cos (\theta / 2)\right)\right] . \tag{4.11}
\end{equation*}
$$

[^5]

Figure 4.1.: Interference of two laser beams. The beams cross under an angle $\theta$ and create a standing light wave in $z$-direction.

According to equation (4.3) the optical lattice potential in $z$-direction is proportional to the light field intensity

$$
\begin{equation*}
V_{l a t}(z)=4 V_{0}^{d i p} \cos ^{2}\left(k_{l a t} z\right)=V_{0} \cos ^{2}\left(\pi z / d_{l a t}\right) \tag{4.12}
\end{equation*}
$$

with $V_{0}$ the lattice depth (four times larger than the dipole trap depth of a single beam), the lattice vector $k_{\text {lat }}=k_{L} \cos (\theta / 2)$ and the period length called lattice spacing

$$
\begin{equation*}
d_{l a t}=\frac{\pi}{k_{l a t}}=\frac{\lambda_{L}}{2 \cos (\theta / 2)} \tag{4.13}
\end{equation*}
$$

If the laser beams are counter propagating $(\theta=0)$ the lattice spacing is equal to half the laser wavelength, $d_{l a t}=\lambda_{L} / 2$.

The lattice depth $V_{0}$ is commonly given in units of the recoil energy that an atom gains by absorption of one photon with momentum $\hbar k_{l a t}$ :

$$
\begin{equation*}
E_{R}=\frac{\hbar^{2} k_{l a t}^{2}}{2 m}=\frac{\hbar^{2} \pi^{2}}{2 m d_{l a t}^{2}} \tag{4.14}
\end{equation*}
$$

For further notations we will use the dimensionless lattice parameter $s=V_{0} / E_{R}$.
A deep one-dimensional lattice can be considered as a stack of highly oblate ODTs, where the radial trapping frequencies are still on the order of a few hundred Hz and the axial trapping frequencies on the single sites are obtained via

$$
\begin{equation*}
\omega_{l a t}=\frac{\pi}{d_{l a t}} \sqrt{\frac{2 V_{0}}{m}}=2 \sqrt{s} \frac{E_{R}}{\hbar} \tag{4.15}
\end{equation*}
$$

with the corresponding lattice oscillator period $T_{l a t}=2 \pi / \omega_{\text {lat }}$ [70, chapter II.B.2].
Like in a solid state body, the particles in each lattice site can tunnel to neighboring sites if the lattice depth is sufficiently small. The tunneling parameter $J$ gives the gain in kinetic energy due to nearest neighbor tunneling. For $V_{0} \gg E_{R}$ the tunneling rate $J / h$ is given by [71]

$$
\begin{equation*}
\frac{J}{h}=\frac{4}{\sqrt{\pi}} \frac{E_{R}}{h} s^{3 / 4} e^{-2 \sqrt{s}} . \tag{4.16}
\end{equation*}
$$

When the tunneling time $h / J$ becomes small compared to experimental time scales (2-10 ms ), particle exchange becomes very small and the on-site wave functions are well-localized within a single lattice site. This corresponds to the so-called tight binding regime.


Figure 4.2.: Setup of the one-dimensional optical lattice and scheme of the experiment in top view. The lattice laser beam (solid red line) is guided through an optical isolator and an AOM to control the beam intensity (used telescopes are not shown). The polarization after the $\lambda / 2$ wave plate is orientated perpendicular to the plane of projection ( $y$-direction). The beam is then focused by a lens onto the atoms, then back reflected under an angle of $\theta \approx 10^{\circ}$ (see figure 4.3) and refocused onto the atomic sample. The atoms are held against gravity (points opposite $y$-direction) with the horizontal ODT1 (dashed red line) and vertical ODT2 (not shown). The magnetic field coils for reaching the Feshbach resonance are shown in green. After a TOF the atoms are recorded via absorption imaging in $x$-direction and imaged by two lenses onto the CCD camera.

### 4.3. Experimental Realisation of the one-dimensional Optical Lattice

In our experiment we use a nearly back reflected configuration of a laser beam that is created by a continuous wave ytterbium fiber laser ${ }^{2}$ at wavelength $\lambda_{L}=1064 \mathrm{~nm}$. The beam is focused onto the atoms with a waist of $72 \mu \mathrm{~m}$, collimated, and back reflected under an angle of $\theta \approx 10^{\circ}$ (see figure 4.2) and refocused onto the atoms. We thus create a one-dimensional optical lattice with spacing $d_{\text {lat }}=534 \mathrm{~nm}$, which is superimposed to the crossed ODT, described in section 5.2.

The fiber laser is used at a power of 8.4 W for the one-dimensional lattice. This laser beam is guided through an optical isolator and an acousto-optic modulator ${ }^{3}$ (AOM) for controlling the laser beam power, as shown in figure 4.2. The laser beam intensity is

[^6]

Figure 4.3.: The required lengths to determine the angle in between the lattice laser beams in top view. The dimensions are given for our vacuum chamber (centered in the figure) and the aluminium rings for mounting the magnetic field coils (right and left side). The trigonometrically calculated angle is $\theta=9.44 \pm 1.32^{\circ}$ and the lattice plain is dumped by $1.9 \pm 1.3^{\circ}$ to the imaging camera.
controlled via an acoustic travelling wave inside the AOM's crystal. The beam is going through the AOM with a diameter of around 1 mm that leads to a rise time of $1 \mu \mathrm{~s}$ because of the finite acoustic velocity within the crystal of the AOM. After these optical components, mirrors and telescopes (not shown in figure 4.2) the polarization direction is aligned in $y$-direction (perpendicular to the plane of incidence) by a $\lambda / 2$ wave plate. The maximum power in the laser beam that creates the optical lattice is around 4.7 W in front of the last lens, before the chamber. The beam is then focused onto the atoms with a waist of $72 \mu \mathrm{~m}$. After this it is back reflected under an angle of around $10^{\circ}$ (see figure 4.2) and refocused onto the atoms using two lenses. During the preparation of the BEC ( $\approx 20 \mathrm{~s}$ ) the shutter is closed and the laser beam switched on to heat the optical components. This technique efficiently reduces temperature effects.

The angle included between the two lattice laser beams was calculated using right triangles where two side lengths were measured. All lengths are shown in figure 4.3 in top view including the dimensions of the vacuum chamber and two aluminium rings for mounting the coils for the magnetic offset field. We determine the angle to be $\theta=9.44 \pm 1.32^{\circ}$, which yields the lattice spacing $d_{l a t}=533.8 \pm 0.5 \mathrm{~nm}$, a re-
coil energy of $E_{R}=13.9 \mathrm{peV} \hat{=} 3.36 \mathrm{kHz} \hat{=} 161 \mathrm{nK}$ and a lattice trapping frequency $\omega_{\text {lat }}=2 \pi \cdot 30 \mathrm{kHz}$ for a depth of $20 E_{R}$, where all the values are calculated for ${ }^{52} \mathrm{Cr}$ with mass $m_{C r}=8.68 \cdot 10^{-26} \mathrm{~kg}$.

### 4.4. Calibration of the Lattice Depth

In experiments with optical lattices, the lattice depth $V_{0}$ (see section 4.2) is defining most of the properties of the system and therefore an important quantity to determine. In principle, the lattice depth can be calculated if one knows all the atomic and lattice parameters. In contrast to the simple two-level Lorentz oscillator model (section 4.1), we have to take into account the coupling of all the energy levels in the spectrum of ${ }^{52} \mathrm{Cr}$. Even though we can perform these calculations, they can only provide a good estimate of the lattice depth. Furthermore, the real laser intensity seen by the atoms is difficult to measure and depends crucially on the alignment of the laser beams.

For experimentally measuring the lattice depth $V_{0}$, we use well-known effects arising in a system of ultracold atoms in periodic potentials. For example Bloch oscillations of a moving condensate in a lattice or Landau-Zener tunneling can be used to calibrate low lattice depth where the tunneling rate is large [72]. However, in our experiments we are mainly interested in larger lattice depths. So we do two independent calibrations of the lattice depth suited for this regime, namely the analysis of interference patterns after time-of-flight (TOF) [72] and the diffraction of atoms from a pulsed optical lattice [73].

### 4.4.1. Interference Patterns - Relative Side-Peak Population

For using the first mentioned calibration method of the lattice depth, the BEC is loaded adiabatically into the lattice (see section 5.2) and then the lattice is switched off suddenly. For a sufficiently deep lattice, the BEC is well described by a sum of Gaussian wave functions localized on all the lattice sites. During a TOF these wave packets will expand freely and interfere with each other and we observe an interference pattern. If the wave packets are still coherent one can see a characteristic pattern of equidistant peaks with spacing $2 \hbar k_{\text {lat }} \cdot t_{T O F} / m$, as shown in figure 4.4a. The coherence is conserved for shallow lattices where tunneling is allowed or for short holding times in the lattice (few ms). The relative side-peak populations $P_{ \pm 1}$ increase with the lattice depth as [72, 74]

$$
\begin{equation*}
s=\frac{16}{\left[\ln \left(P_{ \pm 1}\right)\right]^{2}} P_{ \pm 1}^{-1 / 4} \tag{4.17}
\end{equation*}
$$

This equation can be directly used to calibrate the lattice depth. In the experiment, the interference pattern is averaged over eight pictures for a scattering length of $a=60 a_{0}$ to obtain a high signal-to-noise ratio. This averaging procedure is described in detail in the appendix A.2.3. By fitting a sum of three Gaussian functions, the populations of the zeroth and first orders are extracted. For a lattice depth of $V_{0}=12 E_{R}$ the side-peak


Figure 4.4.: (a) Averaged images of eight shots for increasing lattice beam intensity $I$ in percentage. The images are taken at a scattering length $a=60 a_{0}$ and the field of view for each image is $288.5 \times 129 \mu \mathrm{~m}$. (i) averaged pictures as recorded in the experiment (ii) the thermal cloud is subtraced in every single shot (b) Lattice depth determined via the relative side-peak method against the lattice beam intensity in percentage. The populations $P_{ \pm 1}$ are extracted by a three Gaussian fit from a(ii). The lattice depth is calculated with equation (4.17) and plotted with blue circles with errors coming from the fit. To interpolate for deeper lattices, the lattice depth is fitted by a straight line (solid black line).
population is $P_{ \pm 1}=0.25$ and a second order peak is not visible. Figure 4.4b shows the calibration of the lattice depth for this method against the lattice power. The straight line is a linear fit to the data that yields $V_{0}\left[E_{R}\right]=(1.248 \pm 0.032) I[\%]$. This calibration of the lattice depth is used for the measurements described in chapter 5 .

For our experiment this method is not valid anymore for lattice depths $V_{0}>15 E_{R}$ where tunneling is suppressed. As a result of this, the phases of the separated clouds develop independently and dephasing effects happen as described in [75]. These dephasing effects prevent the application of this calibration method for very deep lattices.

### 4.4.2. Diffraction of a Condensate from an Optical Lattice

A second, independent method for the calibration of the lattice depth $V_{0}$ makes use of an analogon of our system with classical optics: Similar to diffraction of a coherent beam of light from a grating, a BEC (coherent matter wave) is diffracted from an optical lattice for sufficiently short interaction times. These diffraction patterns were first investigated experimentally in 1983 with a beam of sodium atoms [76] and in 1999 with a sodium BEC
[73]. These experiments demonstrated the wave-particle duality and inverted the role of matter and light in the classical double-slit experiment.

When coherent light shines through a grating with spacing $d$, diffraction orders are observable in the far field with spacings $\propto 1 / d$, the light is diffracted by periodic matter grating. For the diffraction of a BEC from an optical lattice, we start with a BEC in the trap (in our case a crossed ODT). Then we abruptly turn on the lattice to a depth $V_{0}$ and hold this value for a variable time $t_{\text {pulse }}$ and then turn it off abruptly ${ }^{4}$ again. The resulting situation is, that a BEC is shortly interacting (compared to the oscillation period) with an optical lattice at constant depth. Immediately after this short interaction time, we use a TOF to let the cloud expand and record the resulting density distribution by resonant absorption imaging. As mentioned in section 2.3.3 a TOF without interactions is similar to a Fourier transformation. The recorded image is therefore displaying the in-trap momentum distribution. We observe several diffraction orders separated by the momentum $2 \hbar k_{\text {lat }}=2 \pi \hbar / d_{\text {lat }}$ and the spacing between neighboring diffraction orders on the recorded image

$$
\begin{equation*}
\Delta z=v_{l a t} \cdot t_{T O F}=\frac{2 \hbar k_{l a t}}{m} \cdot t_{T O F} \tag{4.18}
\end{equation*}
$$

depends on the TOF length $t_{T O F}$. Recalling the analogon to the classical diffraction of a light wave on a refractive index grating, the interaction time $t_{\text {pulse }}$ defines, if the matter wave is diffracted from a 'thin' or 'thick' grating. For a thin grating the interaction time is so short that the displacement of the atoms along the lattice can be neglected. This regime is described in section 4.4.2.1. Whereas for a thick grating the interaction time is comparable to or longer than the oscillation time of the atoms in the standing light wave. This will be discussed in section 4.4.2.2. Hence, for variable interaction times we will expect dynamics of the diffraction patterns.

In figure 4.5 we show an experimental sequence for the dynamics of the population of the diffraction orders with increasing $t_{\text {pulse }}$. In the beginning the maximum observable diffraction order increases linearly with the interaction time. For longer pulse durations there is a boundary given for the maximum diffraction order $n_{\text {max }}$. For increasing further the interaction time the higher order populations decrease until the zeroth order is again the most populated order. We then observe that the pattern starts to repeat again. These revivals are visible for pulse durations that are approximately multiples of $T_{l a t} / 2$ [77]. From the measured maximum order with its momentum $2 \hbar k_{\max }=2 \hbar n_{\max } k_{\text {lat }}$, the lattice depth can be estimated by

$$
\begin{equation*}
V_{0}=\frac{\hbar^{2} k_{\max }^{2}}{2 m}=4 n_{\max }^{2} E_{R} . \tag{4.19}
\end{equation*}
$$

Using this estimation, the lattice depth in figure 4.5 is roughly $V_{0} \approx 256 E_{R}$. A way to measure more precisely the depth of the lattice implies to determine the dynamics of the population of each diffraction order with increasing $t_{\text {pulse }}$. Experimentally, we fit a

[^7]

Figure 4.5.: Atomic density distribution obtained from the diffraction of a BEC by a lattice of depth $225 \pm 9 E_{R}$. The single images are taken for different interaction times $t_{\text {pulse }}$ from $0.2 \mu \mathrm{~s}$ to $6 \mu \mathrm{~s}$. The patterns are recorded after a TOF of 4 ms . The equidistant peaks have a spacing of $2 \hbar k_{\text {lat }} t_{T O F} / m$ and the number of momentum orders is linearly increasing until $2.5 \mu \mathrm{~s}$ and reaches $n_{\max }=8$. The revival point of the zeroth order is visible at $t_{\text {pulse }} \approx 5.6 \mu \mathrm{~s}$. In the images at $5.4 \mu \mathrm{~s}$ and $5.6 \mu \mathrm{~s}$ the zeroth order is splitted in a double peak, which is probably an effect of a slightly misaligned optical lattice. Because of this reason, longer interaction times yield asymmetric diffraction patterns that is not in agreement with theory. The field of view for a single shot is $67.5 \times 1184 \mu \mathrm{~m}$.

Gaussian function to every diffraction order, to extract the normalized populations. Doing this, we 'reconstruct' the dynamics of each order (see figure 4.6), which we compare to theory to finally determine the lattice depth.

The dynamics of the diffraction pattern is well understood and different models have been proposed. It can be described for example by considering the simple case of a twolevel system, as in [78, chapter 4.1 - page 58 ff ] and [79]. In the following however, to be consistent with the picture of the matter waves, we choose to present another description for the dynamics of the diffraction pattern which obtains the same results: in this model, we neglect all atomic interactions during the pulse duration and therefore the system is well described by the Schrödinger equation. This is valid due to the short interaction time $t_{\text {pulse }}$ that has to be much smaller than the radial oscillation time ( $t_{\text {pulse }} \ll T_{r}$ ). In presence of the one-dimensional lattice potential $V_{l a t}=V_{0} \cos ^{2}\left(k_{\text {lat }} x\right)$ (see equation (4.12)), the Schrödinger equation is

$$
\begin{equation*}
i \hbar \dot{\Psi}(x, t)=-\frac{\hbar^{2}}{2 m} \frac{\mathrm{~d}^{2} \Psi(x)}{\mathrm{d} x}+V_{0} \cos ^{2}\left(k_{l a t} x\right) \Psi(x) \tag{4.20}
\end{equation*}
$$

with the normalized matter wave $\Psi(x)$. Starting from this equation we first describe analytical solutions within the so-called Raman-Nath approximation, before numerical methods are presented that solve the full equation (4.20).

### 4.4.2.1. Raman Nath Approximation

For very short pulse durations, the atoms move only by a very small distance compared to the lattice period $d_{l a t}$. Therefore, one can neglect the movement of the atoms during the pulse and hence neglect the kinetic energy term in the Schrödinger equation (first term on right hand side of equation (4.20)). This is called the Raman-Nath approximation. This approximation was first used for the diffraction of light by sound waves in the regime of 'thin' gratings [80, and following parts].
For the optical lattice the pulse durations have to be much smaller than the oscillation time of the atoms on a single lattice site. An upper limit is estimated by the Raman-Nath time $t_{R N}$ [81]

$$
\begin{equation*}
t_{p u l s e}<t_{R N} \equiv \frac{T_{l a t}}{\sqrt{2} \pi} . \tag{4.21}
\end{equation*}
$$

Note that, for the calculations the notation of the pulse duration $t_{\text {pulse }}$ is simplified to $t$. Within the Raman-Nath approximation the kinetic energy in the time-dependent Schrödinger equation is neglected and we obtain a homogeneous differential equation

$$
\begin{equation*}
V_{0} \cos ^{2}\left(k_{l a t} x\right) \Psi(x, t)=i \hbar \dot{\Psi}(x, t) \tag{4.22}
\end{equation*}
$$

The solution of this equation is

$$
\begin{align*}
\Psi(x, t) & =A \cdot e^{-i \frac{V_{0}}{\hbar} \cos ^{2}\left(k_{l a t} x\right) t}=A \cdot e^{-i \frac{V_{0}}{2 \hbar}\left(1+\cos \left(2 k_{l a t} x\right)\right) t} \\
& =A \cdot e^{-i \Omega t} \cdot e^{-i \cos \left(2 k_{l a t} x\right) \Omega t} \tag{4.23}
\end{align*}
$$

with the two photon Rabi frequency $\Omega=V_{0} / 2 \hbar$. These solutions are of course not stationary solutions, because we have neglected the kinetic energy term. To obtain a mathematical description of the measured density distribution after a TOF, we perform the Fourier transformation of the spatial density distribution. For the Fourier transformation we consider only one period length $\pi / k_{\text {lat }}$ of the function (4.23) that leads to the normalization $A=\sqrt{k_{l a t} / \pi}$. Therefore, the momentum distribution writes

$$
\begin{align*}
\mathcal{F} \Psi(x, t)=\tilde{\Psi}(p, t) & =\frac{1}{\sqrt{2 \pi \hbar}} \int_{-\frac{\pi}{2 k_{l a t}}}^{\frac{\pi}{2 k_{l a t}}} \Psi(x, t) \cdot e^{-i \frac{p x}{\hbar}} \mathrm{~d} x \\
& =\sqrt{\frac{k_{l a t}}{2 \pi^{2} \hbar}} \cdot e^{-i \Omega t} \int_{-\frac{\pi}{2 k_{l a t}}}^{\frac{\pi}{2 k_{l a t}}} e^{-i \cos \left(2 k_{l a t} x\right) \Omega t} \cdot e^{-i \frac{p x}{\hbar}} \mathrm{~d} x \tag{4.24}
\end{align*}
$$

using the substitution $z^{\prime}=2 k_{l a t} x+\pi / 2$ leads to

$$
\begin{equation*}
\tilde{\Psi}(p, t)=\sqrt{\frac{1}{8 \pi^{2} \hbar k_{l a t}}} \cdot e^{-i \Omega t} \int_{-\frac{\pi}{2}}^{\frac{3 \pi}{2}} e^{i \sin \left(z^{\prime}\right) \Omega t} \cdot e^{-i \frac{p}{2 \hbar k_{l a t}} z^{\prime}} \cdot e^{-i \frac{p}{2 \hbar k_{l a t}} \frac{\pi}{2}} \mathrm{~d} z^{\prime} \tag{4.25}
\end{equation*}
$$

As mentioned before, we observe diffraction peaks at momenta $p=2 n \hbar k_{\text {lat }}$ only (with $n$ being an integer number). All functions in the integral (4.25) are $2 \pi$-periodic, therefore we are allowed to shift linearly both integration limits. The outcome of this is

$$
\begin{equation*}
\Psi_{n}(t)=\sqrt{\frac{1}{8 \pi^{2} \hbar k_{l a t}}} \cdot e^{-i \Omega t} \int_{-\pi}^{\pi} e^{-i\left(n z^{\prime}-\Omega t \sin \left(z^{\prime}\right)\right)}(-i)^{n} \mathrm{~d} z^{\prime} \tag{4.26}
\end{equation*}
$$

The final equation for the momentum wave function within the Raman Nath approximation is

$$
\begin{equation*}
\Psi_{n}(t)=\sqrt{\frac{1}{2 \hbar k_{l a t}}} \cdot e^{-i \Omega t}(-i)^{n} J_{n}(\Omega t) \tag{4.27}
\end{equation*}
$$

where $J_{n}$ are Bessel functions of the first kind, that are equal to the integral term in equation (4.26) for integer values of $n$ : [82, page 31]

$$
\begin{equation*}
J_{n}(x)=\frac{1}{2 \pi} \int_{-\pi}^{\pi} e^{-\mathrm{i}(n \tau-x \sin \tau)} \mathrm{d} \tau \tag{4.28}
\end{equation*}
$$

The remaining exponential function in (4.27) has no influence on the time propagation as this term is the same for all $n$ (it describes only the global phase evolution). The populations $P_{n}(t)$ of the various diffraction orders are finally given by the square of the absolute values of the wave functions in (4.27)

$$
\begin{equation*}
P_{n}(t)=2 \hbar k_{l a t}\left|\Psi_{n}(t)\right|^{2}=J_{n}^{2}(\Omega t) \tag{4.29}
\end{equation*}
$$

As we will see, our experimental data is in good agreement within the Raman-Nath approximation until reaching the Raman-Nath time $t_{R N}$ (see figure 4.6). However, for our calibration of the lattice depth we use also longer interaction times, going beyond the Raman-Nath regime, which increases the accuracy of the determined lattice depth.


Figure 4.6.: Relative populations of diffraction orders extracted from figure 4.5 are plotted in blue circles against the pulse length $t_{\text {pulse }}$. To determine the lattice depth, the theoretical population in equation (4.38) is fitted to the experimental data (red line). The mean value of the lattice depth (omitting the value from order 8) is $V_{0}=225 \pm 9 E_{R}$. The dashed black line is the theoretical population within the Raman-Nath approximation. It deviates from the data after the time $t_{R N}=2.3 \mu \mathrm{~s}$.

### 4.4.2.2. Beyond the Raman-Nath Regime

To increase the accuracy of the calibration of the lattice depth, we have to measure for times longer than $t_{R N}$. Also the observed revivals of the zeroth order are not predicted within the Raman-Nath approximation. In this regime, the interaction time $t_{\text {pulse }}$ is comparable or longer than the lattice oscillator period $T_{\text {lat }}$ (axial oscillation time). Therefore, the kinetic energy in axial direction is not negligible. We describe this regime again by the Schrödinger equation in presence of an optical lattice potential

$$
\begin{equation*}
E \Psi(x, t)=-\frac{\hbar^{2}}{2 m} \frac{\mathrm{~d}^{2} \Psi(x)}{\mathrm{d} x}+V_{0} \cos ^{2}\left(k_{l a t} x\right) \Psi(x) . \tag{4.30}
\end{equation*}
$$

Using the trigonometric rule $\cos ^{2}(x)=(1+\cos (2 x)) / 2$ and dividing by $E_{R}=\hbar^{2} k_{\text {lat }}^{2} / 2 m$ the result is

$$
\begin{equation*}
\frac{E}{E_{R}} \Psi(x)=-\frac{\mathrm{d}^{2} \Psi(x)}{\mathrm{d} x^{2}} \frac{1}{k_{l a t}^{2}} \Psi(x)+\frac{V_{0}}{2 E_{R}}\left(1+\cos \left(2 k_{l a t} x\right)\right) \Psi(x) . \tag{4.31}
\end{equation*}
$$

Finally using the substitution $z=k_{l a t} x$ it yields to Mathieu's differential equation [83]

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \Psi(z)}{\mathrm{d} z^{2}}+\left[a_{r}-2 q \cos (2 z)\right] \Psi(z)=0 \tag{4.32}
\end{equation*}
$$

with the characteristic value $a_{r}$ and the lattice parameter $q$

$$
\begin{equation*}
a_{r}=\frac{E}{E_{R}}-\frac{V_{0}}{2 E_{R}} \quad \text { and } \quad q=\frac{V_{0}}{4 E_{R}}=\frac{s}{4} . \tag{4.33}
\end{equation*}
$$

The Mathieu equation is found in many physical problems dealing with elliptic cylindrical coordinates or periodic potentials and is well understood [83]. The Mathieu functions $M\left(a_{r}, q, z\right)$ are the corresponding solutions that can be calculated numerically ${ }^{5}$. All Mathieu functions are $2 \pi$-periodic and they are normalized over one period to the value of $\pi$ [83, chapter 2.21]

$$
\begin{equation*}
\int_{-\pi}^{\pi} M\left(a_{r}, q, z\right) \cdot M^{*}\left(a_{r}, q, z\right) \mathrm{d} z=\pi . \tag{4.34}
\end{equation*}
$$

We give a short overview on the properties of Mathieu functions. In the following these properties will be needed to simplify the expressions for the description of the dynamics of the diffraction pattern:

1. For $q=0$ equation (4.32) reduces to $\Psi^{\prime \prime}+a_{r} \Psi=0$ and the solutions are sine and cosine functions. In the case $a_{r} \gg q$ they are still very close to sine and cosine functions [83, chapter 2.11].
2. According to Bloch's theorem any Mathieu function can be written as $e^{i r z} f(z)$ where $f(z)$ is a $2 \pi$-periodic function and $r$ is called the characteristic Mathieu exponent [83, chapter 4.12].

[^8]3. For integer $r$ the Mathieu functions are real and $2 \pi$-periodic. Moreover for odd $r$ they are asymmetric and for even $r$ they are symmetric.
4. For given lattice parameter $q$ and characteristic Mathieu exponent $r$ there is only one possibility for the characteristic value $a_{r}$ to solve the Mathieu equation ${ }^{6}$ (4.32). This solution fulfills the condition $a_{r}>r^{2}$ [83, chapter 2.16].

The Mathieu functions are used to describe the dynamics of the diffraction pattern for long interaction times, where the kinetic energy is non-negligible. To obtain the timedependent populations of the diffraction orders, we calculate the wavefunction of the initial state of the BEC and project it onto the eigenfunctions of the Mathieu equation. These eigenfunctions are then evolved in time with the corresponding eigenenergies up to $t_{\text {pulse }}$ to calculate the wavefunction before the TOF. We finally calculate the Fourier transform to obtain the time-dependent population of the diffraction orders after TOF.

For all the following calculations we consider only one period of the Mathieu functions. The period for all Mathieu functions is $z=[-\pi, \pi]$, which corresponds a spatial interval of two lattice sites. Note that in the diffraction problem, the characteristic Mathieu exponent $r$ is a multiple of 2 . In the following the characteristic Mathieu exponents are $r \equiv 2 j$ with integer $j$. Therefore even Mathieu functions, real and $2 \pi$-periodic, are used (enumeration point 3). The initial BEC wave function is assumed to be constant over a single lattice site during the pulse time and is given by $\Phi(z, t=0)=1 / \sqrt{2 \pi}$. This is valid as long the interaction time is shorter than the radial oscillation time ( $t_{\text {pulse }} \ll T_{r}$ ). When the lattice is pulsed onto the constant BEC wavefunction, the initial state is developed in the even Mathieu functions

$$
\begin{align*}
\Psi(z, t=0) & =\sum_{j=0}^{\infty}\left|M\left(a_{2 j}, q, z\right)\right\rangle\left\langle M\left(a_{2 j}, q, z\right) \mid \Phi(z, t=0)\right\rangle \\
& =\sum_{j=0}^{\infty} c_{2 j} \frac{1}{\sqrt{\pi}} M\left(a_{2 j}, q, z\right) \tag{4.35}
\end{align*}
$$

with the expansion coefficients

$$
\begin{align*}
c_{2 j} & =\left\langle M\left(a_{2 j}, q, z\right) \mid \Phi(z)\right\rangle=\int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} M\left(a_{2 j}, q, z\right) \cdot \frac{1}{\sqrt{2 \pi}} \mathrm{~d} z \\
& =\frac{\sqrt{2}}{\pi} \int_{0}^{\pi} M\left(a_{2 j}, q, z\right) \mathrm{d} z \tag{4.36}
\end{align*}
$$

using equation (4.34) for normalization and the symmetry of even Mathieu functions. As it is not possible to numerically calculate all Mathieu functions up to infinite $j$, we need to truncate the sum in equation (4.35). The corresponding 'truncation criterion' is given

[^9]by the properties of the Mathieu functions. As mentioned before, under the condition $a_{2 j} \gg q$ the even Mathieu functions can be simplified to cosine functions $\cos \left(\sqrt{a_{2 j}} z\right)$ (see enumeration point 1). In this case the expansion coefficients $c_{2 j}$ are zero as the integral $\int_{0}^{\pi} \cos (x) \mathrm{d} x$ vanishes. Therefore, the truncation criterion could be written as $a_{2 j} \gg q$. However, it is more clear to rewrite this truncation criterion as a condition on the maximum number of visible orders $n_{\max }$. Knowing that the characteristic value $a_{2 j}$ is always larger than the squared characteristic exponent $r=2 j$ (enumeration point 4), the truncation criterion can then be written as $16 j^{2} \gg s$, where $s=4 q$ is the dimensionless lattice parameter. $s$ being related to the highest visible diffraction order $s=4 n_{\max }^{2}$, we finally get the truncation criterion $j \gg n_{\text {max }}$. In the calculations we find that the condition $j>n_{\max }+1$ is sufficient.

Taking into account the time evolution operator with eigenenergy $E=\left(a_{2 j}+2 q\right) E_{R}$ from equation (4.33), the time dependent wave function is

$$
\begin{align*}
\Psi\left(z, t_{\text {pulse }}\right) & =\sum_{j=0}^{n_{\text {max }}+1} c_{2 j} \frac{1}{\sqrt{\pi}} M\left(a_{2 j}, q, z\right) e^{-i \frac{a_{2 j}+2 q}{\hbar} E_{R} t_{\text {pulse }}} \\
& =e^{-i \frac{2 q}{\hbar} E_{R} t_{p u l s e}} \sum_{j=0}^{n_{\max }+1} c_{2 j} \frac{1}{\sqrt{\pi}} M\left(a_{2 j}, q, z\right) e^{-i a_{2 j} \omega_{R} t_{p u l s e}} \tag{4.37}
\end{align*}
$$

with the recoil frequency $\omega_{R}=E_{R} / \hbar$. As before in equation (4.27), the first exponential function has no influence for the time evolution of the different populations. Finally we use a Fourier transformation with discrete momenta ${ }^{7} k=2 n$ to obtain the desired momentum distribution

$$
\begin{equation*}
P_{n}\left(q, t_{\text {pulse }}\right)=\left|\int_{0}^{2 \pi} \frac{1}{\sqrt{2 \pi}} \Psi\left(z, t_{\text {pulse }}\right) e^{-2 i n z} \mathrm{~d} z\right|^{2} \tag{4.38}
\end{equation*}
$$

In this final equation the only free parameter is the lattice depth via the parameter $q$, which is used for our calibration of the lattice depth.

The atom numbers in the experimental diffraction orders are fitted using Gaussian functions. From this procedure the relative populations of all visible orders are extracted as a function of pulse length $t_{\text {pulse }}$. For every momentum order, the lattice parameter $q$ is independently fitted with equation (4.38). The semi-automatic evaluation program is described detailed in the Appendix A.2.2. As shown in figure 4.6 we perform nine independent fits and determine precisely the lattice depth to be $V_{0}=225 \pm 9 E_{R}$ (without order 8). The advantage of the Raman-Nath procedure is the short interaction time and therefore it is a good method for calibrating very high lattice depths. Such high lattice depths cannot be calibrated via the interference patterns, where we are limited due to dephasing effects in deep lattices.

Finally, we repeat this calibration procedure for several lattice laser intensities and determine the lattice depth in each case. As the lattice depth is proportional to the

[^10]

Figure 4.7.: Determined lattice depth with the diffraction procedure against the lattice beam intensity $I$ in percentage. Similar to figure 4.6 the lattice depth is extracted with separate fits to the populations of the diffraction orders for several lattice beam intensities. The mean values with the standard deviation (blue circles) is plotted with a linear fit (solid black line) to extrapolate other laser powers.
lattice laser intensity, see equation (4.3), the lattice depth is interpolated by a linear fit. The calibration of the lattice depth used for the measurements described in chapter 5 is shown in figure 4.7. The straight line fit to the laser intensity in percentage is $V_{0}\left[E_{R}\right]=(1.541 \pm 0.040) I[\%]$.

We have shown the calibration of the lattice depth via two independent methods, one through evaluation the interference patterns of a BEC released from the optical lattice, the other via the diffraction of a BEC from the optical lattice. The evaluation of datasets from the same day shows a deviation of the results of around $20 \%$. Up to now it is still unclear which one of the two methods is closer to the true value of the lattice depth. In the following chapter we will use both calibrations in parallel and we will compare the results of our measurements to the full theoretical simulation. There we will see, that the calibration of the lattice depth via the interference patterns leads to better agreement between measurements and theory.

### 4.5. Magnification of the Imaging System

As already mentioned in section 3.1.2 we use absorption imaging for recording the density distribution of the BEC after a TOF. The atoms are imaged via two lenses, as shown in figure 4.2, on a CCD-Chip with a pixel size of $l_{\text {pixel }}=6.45 \mu \mathrm{~m}$. The magnification $M$ of our imaging system is expected to be exactly unity. However, due to the a small uncertainty on the position of the atoms, the real value can deviate slightly from $M=1$.


Figure 4.8.: Using the diffraction of a BEC to calibrate our imaging system (a) Diffraction patterns for a lattice depth $V_{0} \approx 50 E_{R}$ with equal pulse duration $t_{\text {pulse }}=5.5 \mu \mathrm{~s}$ for different expansion times $t_{T O F}$ from 3 ms to 9 ms . The spacing between neighboring orders is increasing linearly in time. The field of view is $104 \times 1227$ $\mu \mathrm{m}$ for each shot (b) Extracted spacing between neighboring orders on the camera for several time of flight lengths $t_{T O F}$ (blue circles). The error bars are inside the point size and therefore not visible. A linear fit (black solid line) gives the velocity on the camera $v_{\text {cam }}$. The ratio of the measured velocity and the theoretical velocity $v_{l a t}=2 \hbar k_{l a t} / m$ yields a magnification factor of $M=1.051 \pm 0.002$ for the imaging. Furthermore, the effective time of flight is $t_{\text {offset }}=0.126 \mathrm{~ms}$ longer than the programmed one.

In any case it is essential to know the scaling of the imaging system to precisely determine the real magnification factor. Only if this value is well known we can rely on the extracted radii and consequently the atom numbers and temperatures.

In the previous section we presented the calibration of the lattice depth via the diffraction patterns. As the separation between the diffraction orders in the measured patterns are well defined by the properties of the optical lattice, one can calibrate the imaging system using this length scale. The spacing $\Delta z$ is given by equation (4.18) and proportional to the expansion time $t_{T O F}$. In the measurements presented in figure 4.8 , the diffraction orders propagate at the constant velocity $v_{l a t}=2 \hbar k_{l a t} / m_{C r}=$ ( $14.301 \pm 0.015) \cdot 10^{-3} \mathrm{~m} / \mathrm{s}$, with uncertainties due to the specified uncertainty of the laser wavelength $\lambda_{L}=1064 \pm 0.5 \mathrm{~nm}$ and the uncertainty on the crossing angle of the laser beams $\theta=9.44 \pm 1.32^{\circ}$.

To extract the experimental 'velocity on the camera' $v_{\text {cam }}$ we pulse the lattice at a constant lattice depth of $V_{0} \approx 50 E_{R}$ and then perform a TOF at variable times $t_{T O F}=$ $3-9 \mathrm{~ms}$ in steps of 1 ms . We use a pulse length of $t_{\text {pulse }}=5.5 \mu \mathrm{~s}$ where the forth diffraction order is clearly observable. One example of this measurement is shown in 4.8 a visualizing the increasing spacing of neighboring orders. This spacing is determined by fitting two Gaussian functions on both two forth orders (left and right) to measure their
distance on the camera. This is repeated for both third and second orders to have an average over nine spacings between neighboring orders for a single TOF length.

The averaged spacings are shown in figure 4.8 for several TOF lengths. A linear fit $v_{\text {cam }} \cdot\left(t_{T O F}+t_{\text {offset }}\right)$ yields a velocity on the camera of $v_{\text {cam }}=(15.033 \pm 0.021) \cdot 10^{-3} \mathrm{~m} / \mathrm{s}$. As mentioned before the diffraction order velocity $v_{\text {lat }}$ is well known and can be used to determine the magnification of the imaging system to $M=1.051 \pm 0.002$. The value required for every information on BEC length scales in our evaluation program is the effective scaling between pixel and real space at the atoms position:

$$
\begin{equation*}
\frac{l_{\text {pixel }}}{M}=\left(6.136 \pm 0.011 \cdot 10^{-6}\right) \frac{\mathrm{m}}{\text { pixel }} \tag{4.39}
\end{equation*}
$$

Furthermore, $t_{\text {offset }}$ yields a $\Delta t_{T O F}=0.1261 \pm 0.0089 \mathrm{~ms}$ longer $t_{T O F}$ than expected from the programmed value.

## 5. Stability of a Dipolar Condensate in a one-dimensional Optical Lattice

In the two previous chapters, we have shown how to obtain a BEC with enhanced dipolar interaction and how to control a one-dimensional lattice. In this chapter, we combine both techniques to investigate the static properties of a dipolar BEC in a one-dimensional optical lattice. In section 5.2 we explain the loading process of the lattice. For a shallow lattice the underlying prolate ODT determines the shape of the BEC. In this geometry the dipolar interaction is mainly attractive and destabilizes the system. On the contrary, for a deep lattice the BEC is sliced into a stack of oblate BECs with long-range dipolar intersite interactions without any particle exchange. In our geometry, the dipolar interaction is on-site repulsive and strongly stabilizes the system, whereas the inter-site interactions are mainly attractive and yield to a destabilization. But still, we will experimentally show in section 5.3 that the overall dipolar interaction can stabilize a many-body quantum system even at attractive contact interaction. The effects of the inter-site interactions are discussed in detail in section 5.4.

### 5.1. A Dipolar Bose-Einstein Condensate in an Optical Lattice

As mentioned in section 2.3 .4 a single oblate dipolar BEC is expected to be stable even at negative scattering length. In our configuration we create for sufficently deep lattices a stack of oblate dipolar BECs (see figure 5.1). In this system, one expects that the mainly attractive inter-site interaction leads to destabilization. Because of these counteracting effects of stabilizing on-site interaction and destabilizing inter-site interaction, the static properties of a dipolar BEC in an optical lattice are not clear and therefore subject of our investigations.


Figure 5.1.: Sketch of a dipolar BEC and a stack of oblate dipolar BECs with spacing $d_{l a t}$


Figure 5.2.: Timeline of our experimental procedure to measure the stability of a chromium BEC. We first create a chromium condensate at high magnetic field ( 600 G ) near the Feshbach resonance. Then the BEC is loaded adiabatically with an s-shaped ramp into the optical lattice. To keep the radial trapping frequencies constant, we decrease the ODT with the same ramp shape. Once the lattice has reached the final depth we tune the scattering length in 6 ms via the magnetic field to its final value. Then we hold the BEC for 2 ms . After this evolution time we release it from the trapping potential to perform a TOF of 6 ms . Finally we image the density distribution by absorption imaging.

### 5.2. Loading a Chromium Condensate in a one-dimensional Optical Lattice

As shown in chapter 3 we perform evaporative cooling to create a BEC in a crossed ODT. During the evaporation the high magnetic field is ramped up to around 600 G near the Feshbach resonance. Then the trapping potential of the crossed ODT is shaped adiabatically to the trapping frequencies $\omega_{x, y, z}=2 \pi \cdot(440,330,290) \mathrm{Hz}$, such that the BEC is still in the ground state of the trapping potential. After this we add the optical lattice along the polarization direction ( $z$-direction). Unlike in section 4.4.2 where the optical lattice is suddenly turned on and several momentum states are occupied, we now aim to load the lattice adiabatically. Therefore, the changing of the lattice depth has to fulfill the conditions for adiabaticity to reach the new ground state of the system. Starting from zero lattice depth the adiabaticity criterion that has to be fulfilled writes [84]

$$
\begin{equation*}
\frac{\mathrm{d} V_{0}}{\mathrm{~d} t} \ll \frac{16 E_{R}^{2}}{\hbar}=16 E_{R} \omega_{R} \tag{5.1}
\end{equation*}
$$

For our experimental values, this criterion implies that the optical lattice should be switched on much slower than $\mathrm{d} V_{0} / \mathrm{d} t \ll 0.3 E_{R} / \mu \mathrm{s}$. To ensure adiabaticity we ramp up the optical lattice to the final lattice depth with an s-shaped ramp ${ }^{1}$ in 20 ms , shown in

[^11]

Figure 5.3.: Checking the adiabaticity for our loading scheme. The first point at $V_{0}=0 E_{R}$ is the case without the overlying lattice. (a) Mean temperature for different lattice depths. S-shaped loading and unloading in 40 ms for different lattice depth. The temperature is constant for all lattice depth that confirms the adiabaticity. The trapping frequencies are $\omega_{x, y, z}=2 \pi \cdot(440,330,290) \mathrm{Hz}$ with the corresponding critical temperature of $T_{c}=540 \mathrm{nK}$ for a system of 40,000 chromium atoms. (b) thermal atom numbers for different lattice depths. The thermal atom numbers are constant for all lattice depths. Together with the constant temperature, we show that the atomic cloud is not heated up during the loading process.
figure 5.2. The ramp is globally slow enough to ensure adiabaticity, and in addition, we make the ramp slower at its beginning, when the lattice depth and therefore the energy spacing to the first excited state are small. Ramping up the lattice laser beam, we also add some extra radial confinement. Additionally in our measurement, we want to keep the radial frequencies constant, and therefore the power of the horizontal ODT beam is decreased also s-shaped (see figure 5.2).

Experimentally we check the adiabaticity, by loading the BEC into the optical lattice and immediately decrease the lattice intensity with the same ramp inversed. Then we perform a 6 ms TOF and record the atoms by absorption imaging. The initial BEC is chosen not to be purely condensed with a remaining thermal cloud. This allows us to measure the temperature of the cloud. If our loading ramp is adiabatically we should not heat the system and the thermal atom number should not increase.

From the recorded density distribution, we extract the thermal fraction. The thermal density distribution in a harmonic trap has a Gaussian shape with widths $l_{i}$, which we extract from a bimodal fit to the cold cloud. The corresponding temperature of the


Figure 5.4.: Lifetime in a deep lattice ( $V_{0}=41.6 \pm 1.7 E_{R}$ ) for different scattering lengths a. For being further away from the Feshbach resonance and the instability point $\left(a_{\text {crit }} \approx-15 E_{R}\right)$ the $1 / \mathrm{e}$ lifetime is increasing. It stays always below the lifetime of a dipolar BEC in a crossed ODT and above our measurements timescale of 2 ms.
thermal cloud after an expansion is given by [57]

$$
\begin{equation*}
T=\frac{m}{k_{B}} \frac{l_{i}^{2}\left(t_{T O F}\right)}{\frac{1}{\omega_{i}^{2}}+t_{T O F}^{2}} \tag{5.2}
\end{equation*}
$$

with $l_{i}\left(t_{T O F}\right)$ the widths after an expansion time $t_{T O F}$. The atom mean temperature (averaged for at least five images) is shown for different lattice depths in figure 5.3. The temperature is constant for every lattice depth, so there is no visible heating effect due to the lattice loading procedure. In addition the number of thermal atoms does not increase after loading and unloading the lattice. Therefore we show that we are able to load adiabatically a chromium BEC into a one-dimensional lattice up to depths $V_{0}=90 E_{R}$.

## Lifetime in the Optical Lattice

The lifetime of a BEC is strongly reduced when going close to the Feshbach resonance due to inelastic 3 -body losses. The 1 /e lifetime of a chromium BEC in a crossed ODT in the vicinity of the used Feshbach resonance was already measured in [10]. The lifetime far away from the resonance was longer than 1 s , whereas near the resonance the lifetime decreased to 40 ms due to inelastic losses. So it is not clear if the lifetimes are long enough for measurements when using an optical lattice, as we expect high densities which lead to an enhancement of 3 -body losses.

Therefore, we have done lifetime measurements in a deep lattice $\left(V_{0}=42 E_{R}\right)$ at low and even negative scattering length. We create a BEC in high magnetic field, load it into the lattice as described before, then tune the scattering length to the final value and hold
for variable time $t_{\text {hold }}$ (see figure 5.2). For each $t_{\text {hold }}$ we measure the number of atoms in the condensate and we fit this decaying curve by an exponential function to extract the $1 / \mathrm{e}$ lifetime. This lifetime as a function of the scattering length $a$ is shown in figure 5.4. The lifetime of the BEC in the lattice is below 40 ms and thus shorter than for a BEC trapped in the crossed ODT only. As expected the lifetime is shorter due to the higher mean frequency with superimposed lattice. For a lattice depth of $V_{0}=42 E_{R}$ the mean frequency is around $\bar{\omega}=2 \pi \cdot 1800 \mathrm{~Hz}$, while $\bar{\omega}$ is much lower for a BEC in the crossed ODT ( $\bar{\omega} \approx 2 \pi \cdot 300 \mathrm{~Hz}$ ).
For the interesting range of scattering length, namely $a<a_{d d} \approx 15 a_{0}$ the lifetime is sufficiently long for the stability measurements, as it should be longer than the holding time of 2 ms at the final scattering length.

### 5.3. Stability Diagram of a Chromium Condensate in a one-dimensional Lattice

Now, that we are able to adiabatically load the dipolar BEC into the optical lattice, we are able to investigate the static properties of the system. The procedure is similar to the one in [11] for the measurements shown in section 2.3.4. In detail, the experimental procedure is shown in figure 5.2 and is as follows: the chromium BEC is produced in high magnetic field at a scattering length of $a \approx 90 a_{0}$ in a crossed ODT with trapping frequencies $\omega_{x, y, z}=2 \pi \cdot(440,330,290) \mathrm{Hz}$. The dipoles are aligned along the strong magnetic field in $z$-direction. After reducing the scattering length to $a=60 a_{0}$, we load the BEC into the one-dimensional optical lattice, which is oriented along the polarization axis in $z$ direction. We populate around 15 lattice sites, the central sites containing around 2000 atoms. Then we decrease the scattering length in 6 ms to its final value and hold the atoms for $t_{\text {hold }}=2 \mathrm{~ms}$. Finally we perform a TOF of $t_{\text {TOF }}=6 \mathrm{~ms}$ and take an absorption image.

The BEC atom number is extracted by a 1D bimodal fit. We explain the fitting procedure in detail in appendix A.2.1. The initial BEC atom number before loading into the lattice is $N \approx 20,000$. For large scattering lengths, we measure typically $10,000 \mathrm{BEC}$ atoms in a deep lattice and $15,000 \mathrm{BEC}$ atoms in a shallow lattice. Getting close to the instability point the BEC atom number abruptly decreases as shown in figure 5.5. By fitting an empirically chosen function ${ }^{2}$ to the observed BEC atom numbers, we find the critical scattering length $a_{\text {crit }}$. For a shallow lattice around $V_{0}=7 E_{R}$ the critical scattering length is positive $a_{\text {crit }}=6.5 \pm 1.9 a_{0}$. For deeper lattices around $V_{0}=45 E_{R}$ the scattering length is negative $a_{\text {crit }}=-13.2 \pm 2.5 a_{0}$.

In figure 5.6 we show the stability diagram of a chromium BEC in a one-dimensional lattice, which is obtained by measuring $a_{\text {crit }}$ for several lattice depths $V_{0}=0-80 E_{R}$. The critical scattering length decreases continuously for increasing lattice depth. $a_{\text {crit }}$ is

[^12]

Figure 5.5.: Atom numbers versus scattering length for a shallow lattice (blue circles) and deep lattice (filled red circles). The solid lines are fits to the data using an empirically chosen function $N=\max \left[N_{0}\left(a-a_{\text {crit }}\right)^{\beta}, 0\right]$ from which we extract the critical scattering length $a_{\text {crit }}$.
positive until $V_{0} \approx 10 E_{R}$ and in a deep lattice it goes down to $a_{\text {crit }}=-17 \pm 3 a_{0}$.
The solid red line in figure 5.6 is obtained from numerical simulations based on the nonlocal GPE (equation (2.26)) using exactly our experimental parameters. The simulations agree well with the experimentally determined stability threshold. The simulations were done on a full 3D numerical grid in imaginary time to find the ground state of the system, which means no assumptions were made on the wave function (in contrast to e.g. the Gaussian ansatz in section 5.4). Alternatively, the non-local GPE was evaluated in real time and simulated exactly our experiment as described above. Both results of the critical scattering length are basically undistinguishable. This implies that the experimental results recover the stability threshold even for deep lattices, where reaching the ground state is questionable, due to the suppression of tunneling.

As our BEC atom number varies during the measurement day ( $\Delta N \approx 5000$ ), we checked the theoretical dependence of $a_{\text {crit }}$ on $N$. The black dashed and dotted lines are full simulations for initial BEC atom numbers of 30,000 and 10,000 . They only show a weak dependence on the atom number, especially for deep lattices. To show the difference from a purely contact interacting BEC the experiment is simulated without magnetic dipole moment with chromium parameters otherwise. This is shown as the green dashed-dotted line and the critical scattering length for the whole range is $\left|a_{\text {crit }}\right|<0.4 a_{0}$.

In the absence of the lattice, the ODT determines the shape of the dipolar BEC and for the chosen trapping frequencies the BEC is prolate. The BEC is destabilized by attractive dipolar interaction and becomes unstable for positive scattering lengths close


Figure 5.6.: Stability diagram of a dipolar condensate in the one-dimensional lattice. The critical scattering length $a_{\text {crit }}$ is plotted against the lattice depth $V_{0}$. The lines are numerical simulations based on the non-local GPE for different atom numbers. For increasing lattice depth we observe a crossover from a dipolar destabilized $\left(a_{\text {crit }}>0\right)$ to a dipolar stabilized $\left(a_{\text {crit }}<0\right)$ regime. The green dashed dotted line is the simulation for the purely contact interacting case with otherwise chromium parameters. The Lattice depth is calibrated in two independent ways (a) via the diffraction method (Raman-Nath) and (b) with the relative side-peak method (interference).
to the value $a_{d d}=15 a_{0}[11]$. For lattice depths smaller than $V_{0}<12 E_{R}$, the stability depends on the interplay of the interactions and the tunneling rate in the lattice and is very hard to describe. For deeper lattices $\left(V_{0}>12 E_{R}\right)$ tunneling is suppressed and the prolate BEC is sliced into a stack of highly oblate dipolar BECs. The oblate dipolar BECs are even stable for negative scattering lengths. For increasing lattice depth we observe a crossover from a dipolar destabilized to a dipolar stabilized regime. This means the stabilization due to the large condensate aspect ratio is stronger than any possible intersite destabilization mechanism. The inter-site interaction without particle interaction is particular to long-range interacting systems, and we will investigate them further in the next section 5.4.

As mentioned in section 4.4 we have two independent ways to calibrate the lattice depth. In figure 5.6a the diffraction method (Raman-Nath) is used, whereas in figure 5.6b the interference patterns with the relative side-peak population were chosen as calibration method. We find a better agreement of the numerical solution with the measurements for the lattice calibration via the relative side-peak population. A possible explanation for this result is: we use the Raman-Nath diffraction method in low magnetic field and pulse the lattice immediately after releasing the BEC from the ODT. In contrast to this, we extract
the relative side-peak population in high magnetic field from interference patterns at 60 $a_{0}$ with the same external trapping as we use for the stability measurements. Cosequently, it is performed exactly under the same conditions present in the stability measurements. Therefore, the calibration of the lattice depth via the interference patterns is probably closer to the true value.

### 5.4. Dipolar inter-site Interactions in a one-dimensional Optical Lattice

As tunneling is suppressed in the deep lattice regime the system is well described by independent wavefunctions, localized on the sites of the optical lattice. Tunneling is negligible for $V_{0} \geq 12 E_{R}$ where the tunneling rate is $J / h \leq 50 \mathrm{~Hz}$ that leads to a tunneling time of $h / J \geq 20 \mathrm{~ms}$. This is ten times larger than the holding time at the final scattering length of $t_{\text {hold }}=2 \mathrm{~ms}$. In this tight-binding model the several site populations do not change due to negligible particle exchange. Therefore, one can assume the system to be a stack of oblate BECs, that are coupled only via the long-range dipolar inter-site interactions.

To investigate the role of the inter-site interactions on the stability threshold, we repeat the numerical simulations without inter-site interactions. This is done by introducing a truncated dipolar potential $U_{d d}^{b o x}(\boldsymbol{r})=U_{d d}(\boldsymbol{r})\left[\Theta\left(\left(\boldsymbol{r} \cdot \hat{\boldsymbol{e}}_{z}\right)+d_{l a t} / 2\right)-\Theta\left(\left(\boldsymbol{r} \cdot \hat{\boldsymbol{e}}_{z}\right)-d_{l a t} / 2\right)\right]$, where $\Theta(x)$ is the Heaviside function. With this potential only the dipolar and contact on-site interactions are taken into account, whereas the dipolar inter-site interactions are effectively removed. Figure 5.8 shows the deep lattice regime of the stability diagram for the full and the truncated potential simulations. The simulation with the truncated potential shows a large deviation from the experimental data in the full range of measurements. As an exmaple, at $V_{0} \approx 18 E_{R}$ this deviation is around $8 a_{0}$ which is three times larger than our standard deviation. This discrepancy, in addition to the good agreement with the full simulation, confirms the presence of destabilizing dipolar inter-site interactions.

Numerical simulations give very exact results without any assumption on the wavefunction, but one data point takes several hours for these calculations. Hence, for qualitative understanding, we assume a fixed shape for the wavefunctions for faster analytic calculations. We approximate the on-site wavefunctions by radially symmetric Gaussian functions, which is a good approximation in the tight-binding model. To find the stable regions of a coupled stack of oblate BECs we search for minima in the energy functional having the BEC radii as variational parameters $\sigma_{r}$ and $\sigma_{z}$, similar to the procedure described in section 2.3.4. For the Gaussian ansatz the on-site energy terms are given in the equations (2.35)-(2.37). The on-site energy consists of the quantum pressure term, the potential energy, the contact and the dipolar interaction energy. For a stack of coupled dipolar BECs we also have to take into account the inter-site interaction energy. For simplicity, we assume equal atom numbers on $N_{l a t}$ lattice sites. Each lattice site with spacing $d_{\text {lat }}$ contains $N$ atoms in the BEC and the spacing between the sites is denoted by $d_{l a t}$


Figure 5.7.: The effects of the inter-site interaction is calculated for different numbers of filled lattice sites $N_{\text {lat }}$ with distance $d_{\text {lat }}$. Each oblate dipolar BEC contains $N=2000$ atoms. The case $N_{l a t}=1$ considers only on-site interaction. For $N_{l a t} \geq 2$ each inter-site interaction to the $j$-th nearest neighbor is calculated.
(see figure 5.7). For the $j$-th nearest neighbors with polarization in lattice direction the inter-site interaction energy is given by [17]

$$
\begin{equation*}
\frac{E_{\text {Inter }}^{j}}{\hbar \bar{\omega}}=\sqrt{\frac{2}{\pi}} \frac{N^{2} a_{d d}}{a_{h o}} \frac{1}{\sigma_{r}^{3}} F\left(c, \frac{j d_{l a t}}{a_{h o} \sigma_{r}}\right) \tag{5.3}
\end{equation*}
$$

where $c=\sqrt{1-\sigma_{z}^{2} / \sigma_{r}^{2}}=\sqrt{1-1 / \kappa^{2}}$ and

$$
\begin{equation*}
F(u, v)=\int_{0}^{1} \frac{3 s^{2}-1}{\left(1-u^{2} s^{2}\right)^{3 / 2}}\left(1-\frac{v^{2} s^{2}}{1-u^{2} s^{2}}\right) e^{-\frac{v^{2} s^{2}}{2\left(1-u^{2} s^{2}\right)}} \mathrm{d} s . \tag{5.4}
\end{equation*}
$$

Using these on-site and inter-site energies the total energy is a function of the variational parameters $\sigma_{r}$ and $\sigma_{z}$. To obtain the total system energy, the on-site energy is multiplied by $N_{l a t}$ and the inter-site energy with the $j$-th nearest neighbor is multiplied by $N_{l a t}-j$ to account for the finite size of the system. Similar to section 2.3.4, the presence of a positive global or local minimum corresponds to the presence of a stable ground state. Note that the prefactor of the inter-site energy is doubled compared to the on-site energy contact interaction (see equation 2.37), as we count it only once for each pair of coupled BECs.
With this Gaussian ansatz we will first investigate the situation of the stability measurements. In the experiment, we load around 15 lattice sites with the atoms parabolic distributed with 2000 atoms in the central sites. For the first set of calculations, we will use the following simplified picture: uniformly distributed atoms on 15 lattice sites with a relative distance of $d_{l a t}=534 \mathrm{~nm}$, each lattice site containing 2000 atoms. The trapping frequency in $z$-direction is defined by the lattice (equation (4.12)). As the Gaussian ansatz is radially symmetric, we have to consider a radially symmetric trapping potential. For this we take the mean radial frequency $\omega_{r}=\sqrt{\omega_{x} \omega_{y}}=2 \pi \cdot 380 \mathrm{~Hz}$. For our experimental values this leads to trap aspect ratios of $\lambda=55$ for a lattice depth of $V_{0}=10 E_{R}$ and $\lambda=175$ for a lattice depth of $V_{0}=100 E_{R}$.

The results for the critical scattering length are shown in figure 5.8. The blue line is the result of the system described above with 15 lattice sites filled. The blue dashed line


Figure 5.8.: Zoom in figure 5.6 for the deep lattice regime. The full numerical simulation (red solid line) and the numerical simulation without inter-site interaction (red dashed line) are given. The calculations without the inter-site interaction show a substantial deviation to the experimental data, indicating the presence of destabilizing dipolar inter-site interactions. In addition, the analytical simulation with the Gaussian ansatz is given for a single dipolar condensate (blue dashed line) and 15 equal coupled condensates (blue solid line) each with 2000 atoms.
is resulting for only a single filled lattice site $N_{l a t}=1$. This is equivalent to removing inter-site interactions comparable to the use of the truncated potential explained above. The resulting stability threshold matches the experimental datapoints within the error bars, therefore providing an appropriate theoretical description of the system. When comparing the simulation and the Gaussian ansatz for $N_{l a t}=1$, the stability threshold using the Gaussian ansatz is smaller than the numerical simulation in the whole range. The same case occurs for $N_{l a t}=15$ in the experimentally accessible range.

As the density distribution is forced to a Gaussian shape the maximum density is always in the center of the trap. In this case the BEC shrinks in radial size for scattering lengths approaching $a_{\text {crit }}$. Because here the whole BEC is contracting towards the center, leading to an 'implosion' of the condensate, this mechanism is called 'global collapse'. When doing the full simulations the density distribution is not predefined and local density maxima off the trap center can occur. In certain configurations, the instability then is induced at these locations off the center [85]. This 'local collapse' leads to a less stable BEC compared to simulations with the Gaussian ansatz. Nevertheless, the Gaussian ansatz describes the role of the inter-site interaction remarkably well. For example at $V_{0}=18 E_{R}$ both simulations yield the same destabilization effect of the inter-site interaction $\Delta a_{\text {crit }} \approx 8 a_{0}$.

### 5.5. Interactions between spatially separated Dipolar Condensates

In the previous section we kept the radial trapping frequency $\omega_{r}$ constant and controlled the axial frequency $\omega_{z}$ by the lattice depth $V_{0}$. However, the mean trapping frequency $\bar{\omega}=$ $\left(\omega_{r}^{2} \omega_{z}\right)^{1 / 3}$ is not constant for the experimental measurement. For further investigations of the effects coming from the inter-site interactions, we will study a different system with constant mean trapping frequency $\bar{\omega}$. Hence, we only change the trapping geometry with the value $\lambda$ the trap aspect ratio. Unless explicitly stated, in the following we will consider the system:

- $N_{\text {lat }}$ lattice sites
- $N=2000$ atoms per lattice site
- lattice spacing $d_{l a t}=534 \mathrm{~nm}$
- mean trapping frequency of $\bar{\omega}=2 \pi \cdot 1500 \mathrm{~Hz}$ (mean harmonic oscillator length $a_{h o}=359 \mathrm{~nm}$ )

We calculate the critical scattering length, the interaction energy and the radial size of the condensate using the Gaussian ansatz for the wavefunction as in the previous section 5.4.

### 5.5.1. Critical Scattering Length for coupled Dipolar Condensates

We investigate the dependency of the critical scattering length on the number of neighboring filled lattice sites using the Gaussian ansatz. As can be seen in figure 5.9, the most stable system is the single BEC, stabilized by repulsive dipolar interactions. Adding more lattice sites destabilizes the system meaning that the inter-site interactions are mainly attractive. One can infer an 'asymptotic' stability diagram by increasing the number of lattice sites to very large values: for $N_{l a t}=200$, the critical scattering length $a_{\text {crit }}$ differs only by around $0.5 a_{0}$ from the system with 50 lattice sites. Experimentally the number of filled lattice sites is $N_{l a t}=15$ which varies only $1.5 a_{0}$ from the 'asymptotic' value in the investigated regime.

In addition, we compare the calculated stability threshold of a dipolar BEC to calculations where we set the dipole moment to zero (i.e. purely contact interacting system with chromium parameters otherwise). Here the critical scattering length is negative as well but much closer to the zero value. In the full range between $\lambda=10-1000$ the minimum is $a_{\text {crit }}=-2 a_{0}$. The negative scattering length is stabilized by the quantum pressure and the external trapping.


Figure 5.9.: Critical scattering length $a_{\text {crit }}$ against trap aspect ratio $\lambda$ for different number of lattice sites $N_{\text {lat }}$. The single BEC (lower blue solid line) is the most stable system. Filling more lattice sites destabilizes the system up to the 'asymptotic' value (upper solid blue line). Red dashed lines are the asymptotic values for a prolate trap $\left(a_{d d}\right)$ and highly oblate trap $\left(-2 a_{d d}\right)$. The green curve marks the stability threshold for a purely contact interacting BEC with otherwise the same parameters.

### 5.5.2. Inter-site Interaction Energy

In this section we calculate the inter-site interaction energy, which turns out to be negative between all the lattice sites in our system. We consider a purely dipolar BEC at $a=0 a_{0}$ (without any contact interaction). We search for the ground state of a system with $N_{l a t}=200$ lattice sites. From this system we extract the inter-site interaction energy for two separated oblate BECs with spatial distance $d=j d_{l a t}(j \in \mathbb{N})$. The inter-site interaction energy is shown for different distances in figure 5.10 for different trap aspect ratios $\lambda$. For each distance, lattice site and trap aspect ratio the inter-site interaction energy is negative and consequently attractive. A repulsive interaction energy is only possible for overlapping Gaussians, which is not observable in the tight binding regime. The inter-site interaction energy decreases for higher trap aspect ratios. If the trap aspect ratio is increased the BEC grows in the radial direction.

The asymptotic value of the inter-site energy of two infinite parallel homogeneous dipolar discs is zero [86, 87]. This can be shown with the following calculation: the interaction


Figure 5.10.: Inter-site interaction energy for a dipolar BEC versus spatial distance. The ground state of 200 lattice sites with each 2000 atoms and spacing $d_{l a t}=534 \mathrm{~nm}$ at the scattering length $a=0 a_{0}$ is considered. The interaction energy (in frequency) of a single site with a second site with spatial distance $d$ is shown. For every distance the interaction is attractive. (a) inter-site interaction energy with linear scale (b) the same data in log-log scale, where for large distances the interaction energy is proportional to $1 / r^{3}$ for all systems with different trap aspect ratio $\lambda$.
potential between two magnetic dipoles belonging to two different lattice sites is [87]

$$
\begin{equation*}
U_{\text {inter }}(r)=\left(\frac{\mu_{0} \mu^{2}}{4 \pi}\right)^{2} \frac{r^{2}-2 d_{l a t}^{2}}{\left(r^{2}+d_{l a t}^{2}\right)^{5 / 2}} \tag{5.5}
\end{equation*}
$$

where $r$ is the in-plane separation between these dipoles. We now consider a single dipole interacting with a two-dimensional disc with radius $R$ with a homogeneous dipole density. In this case, the inter-site interaction is proportional to

$$
\begin{equation*}
E_{\text {int }}(R) \propto \int_{0}^{R} r \cdot U_{\text {inter }}(r) \mathrm{d} r \propto-\frac{R^{2}}{\left(R^{2}+d_{\text {lat }}\right)^{3 / 2}} \tag{5.6}
\end{equation*}
$$

Therefore, the interaction energy between a single dipole and an infinite extended homogeneous disc is zero. As the problem is radially symmetric, the interaction energy for two infinite homogeneous discs is also zero.

In figure 5.10b the inter-site energy is plotted in a log-log scale. For large distances (in this case more than $40 d_{l a t}$ ) the inter-action energy is proportional to $1 / r^{3}$. For this reason, systems at any trap aspect ratios $\lambda$ can be described as two interacting point-like magnetic dipoles, provided the distance between them is sufficiently large.


Figure 5.11.: Radial cloud size (using Gaussian ansatz) against the scattering length $a$ for several number of filled lattice sites $N_{l a t}$. A system with $N_{\text {lat }}$ sites shrinks radially for decreasing the scattering length and at $a_{\text {crit }}$ its radial collapse takes place. For adding more neighbors the radial size shrinks at a constant scattering length. For smaller radial size the absolute value of the inter-site energy is higher and the system gets more unstable. (a) for a trap aspect ratio $\lambda=70$ (b) for a very oblate trap aspect ratio $\lambda=1000$, where the effect of increasing number filled lattice sites is smaller compared to $\lambda=70$.

### 5.5.3. Radial Size modified by inter-site Interactions

When performing variational calculations to find the stability threshold of a single BEC, we have seen that the condensate contracts radially until it reaches zero size and the collapse occurs. Hence, the radial size is an interesting observable, which we examine in presence of inter-site interactions in this section. Using the Gaussian ansatz we consider a system with different number of filled lattice sites $N_{l a t}$. We calculate the ground state for different scattering lengths and for two different trap aspect ratios. The radial size of the ground state is shown in figure 5.11 for different number of lattice sites against the scattering length $a$ close to $a_{\text {crit }}$. The calculations are carried out for two traps: for a trap aspect ratio of $\lambda=70$ that is experimentally created at a lattice depth of $V_{0}=15 E_{R}$ and a very oblate $\operatorname{trap} \lambda=1000$ that is close to an infinitely thin oblate BEC.

First, we consider the case of a single BEC $N_{l a t}=1$. The BEC shrinks radially for decreasing scattering length until it collapses at $a=a_{\text {crit }}$. A similar radial collapse is also found when several sites are populated, see figure 5.11.

In presence of inter-site interactions, the system shrinks radially at constant scattering length. This effect can be understood, when recalling that two point-like dipoles on neighboring lattice sites are attracting each other (lower their potential energy), whereas


Figure 5.12.: Radial cloud size (using Gaussian ansatz) against the effective scattering length $a_{\text {eff }}$ for several number of filled lattice sites $N_{l a t} . a_{\text {eff }}$ is defined such that all systems with different $N_{l a t}$ collapse at $a_{\text {eff,crit }}=0 a_{0}$ (it is a linear transformation like $a_{\text {eff }}=a-a_{\text {crit }}\left(N_{\text {lat }}\right)$ compared to figure 5.11). Then one can see the unitary radial collapse for all systems of $N_{l a t}$. In green it is shown the case for a contact interacting BEC with otherwise same parameters as chromium. This case is for every scattering length smaller than the dipolar case. (a) for a trap aspect ratio $\lambda=70$ (b) for a trap aspect ratio $\lambda=70$.
two infinite planes of dipoles are not interacting at all. This effect is more prominent in moderately oblate traps than in extremely oblate traps, see comparison between $\lambda=70$ and $\lambda=1000$ trap in figure 5.11.

The behaviour of the radial size against the scattering length for different number of lattice sites $N_{\text {lat }}$ is qualitatively the same in all cases. Therefore, they are compared in the following way: We define an effective scattering length where all the systems collapse at $a_{\text {eff,crit }}=0 a_{0}$. To fulfill this definition we shift all curves linearly by using the transformation $a_{\text {eff }}=a-a_{\text {crit }}\left(N_{l a t}\right)$. As a result all curves overlap at their instability point, see figure 5.12. As the curves almost overlap in the full range of $a_{\text {eff }}$, the inter-site interactions essentially shift the critical scattering length, while leaving the qualitative behaviour unchanged. However, we have to keep in mind our simplification of the system due to the Gaussian ansatz. Several theoretical investigations have revealed interesting qualitative effects as well [32, 88].


Figure 5.13.: Total inter-site interaction energy for a dipolar BEC with spatial distance for several trap aspect ratios $\lambda$. We consider a purely dipolar BEC with 20,000 atoms at $a=0 a_{0}$. These atoms are uniformly distributed on $N_{l a t}$ lattice sites and we calculate the ground state. (a) The total inter-site interaction energy of the whole system against the number of filled lattice sites $N_{l a t}$. The calculations are done for different trap aspect ratios $\lambda$. The absolute value of inter-site interaction energy is maximized for low trap aspect ratios. The red dots mark the maximum absolute value of the inter-site interaction of each system with trap aspect ratio $\lambda$. (b) The total energy of the system (on-site and inter-site) against different number of filled lattice sites. For small trap aspect ratios the total energy is lower. A minimum for a single system with trap aspect ratio $\lambda$ is not visible.

### 5.5.4. Maximum inter-site Energy of a Dipolar Condensate

In the previous sections we fixed the number of atoms in all lattice sites to $N=2000$, independent on the number of total filled lattice sites $N_{l a t}$. However, in our experiment we have a limited total number of atoms (typically 20,000) than can be distributed on $N_{l a t}$ lattice sites. Hence, we check at which number of loaded lattice sites we find maximum inter-site interactions. Furthermore we want to know the system with the relative highest inter-site interaction energy compared to the total energy of the system (on-site and inter-site energy).

For this we consider a purely dipolar BEC with 20,000 atoms at $a=0 a_{0}$ with no influence of contact interaction. To mimic our experiment we uniformly distribute the 20,000 atoms on $N_{\text {lat }}$ lattice sites. Note that in this system the atom number per site is $N=20000 / N_{l a t}$, in contrast to the sections before. We extract the inter-site interaction energy and the total energy of the whole system, for several lattice sites $N_{l a t}$ and trap aspect ratios $\lambda$.

The results are shown in figure 5.13. Figure 5.13a shows the total inter-site interaction against the number of filled lattice sites. For moderate trap aspect ratios the total intersite interaction reaches a maximum value. The red dots mark the maximum absolute value of the inter-site interaction of each system with trap aspect ratio $\lambda$. The value with maximum inter-site interaction is for the whole investigated range of trap aspect ratios between $N_{\text {lat }}=9-18$. This regime is experimentally accessible as we load 20,000 chromium atoms in 15 lattice sites for the stability measurements.
Figure 5.13 b shows the total energy of the considered system. Even though the inter-site interaction energy has a clear minimum between $N_{l a t}=9-18$, the total energy does not show such behaviour, due to the interplay between on-site and inter-site interactions. Instead the total energy is nearly constant for sufficient high number of loaded lattice sites. Therefore, the relative total inter-site interaction reaches the maximum for small trap aspect ratios

## 6. Conclusion and Outlook

## Conclusion

In this thesis we have presented the experimental investigation of the static properties of a dipolar BEC in a one-dimensional optical lattice. We have reached the stable regime of highly oblate dipolar BECs with attractive contact interaction, where several unique physical phenomena are predicted.

Such experiment implied to produce a dipolar BEC and to know its basic properties, which we introduced in the first two chapters. It was also essential to control the interactions inside the BEC and to handle a one-dimensional optical lattice. Furthermore the experiments required a very high level of control on two important parameters: first the scattering length $a$, describing the contact interaction strength and tunable by a magnetic Feshbach resonance, and second the lattice depth $V_{0}$, describing the properties of the optical lattice potential and that can be tuned by changing the intensity of the laser beams creating it. A major effort was made to calibrate both parameters as accurately as possible.

These requirements being fulfilled, we studied the stability of a chromium BEC in the one-dimensional optical lattice. The stability threshold depends on the complicated interplay between tunneling, contact interaction and dipolar interactions. However, in the deep lattice regime, tunneling between the lattice sites becomes negligible and the system can be considered as a stack of highly oblate dipolar condensates. In this regime, we have shown the dipolar stabilization of an otherwise unstable many-body system such as an attractive Bose gas. The mainly repulsive dipolar interaction compensates the destabilizing attractive contact interaction and stabilizes the system up to negative scattering lengths of $a=-17 a_{0}$. In contrast, for reduced lattice depths the dipolar interaction becomes mainly attractive and destabilizes the system. The tunable parameters (scattering length $a$ and lattice depth $V_{0}$ ) being well controlled and well calibrated, our measurements are in excellent agreement with numerical simulations without any adjustable parameter.

To understand the behaviour of the stability threshold for a stack of oblate dipolar condensates, the long-range dipolar interaction between the single BECs (inter-site interactions) have to be taken in account. For this we developed a model using the Gaussian ansatz for the wavefunction and we quantified the destabilization induced by inter-site interactions.


Figure 6.1.: Excitation spectra of highly oblate dipolar BECs. Curves calculated for following values: $\omega_{z}=2 \pi \cdot 42 \mathrm{kHz}$ (corresponding to $V_{0}=40 E_{R}$ ), axial size $l_{z}=\sqrt{\hbar / m \omega_{z}}=$ 68 nm , density $n=10^{15} \mathrm{~cm}^{-3}$ and lattice spacing $d_{\text {lat }}=534 \mathrm{~nm}$ (a) single dipolar BEC at three different scattering lengths (b) for a single site (dashed line) and two sites (solid lines) (c) single site (dashed line) and 10 sites.

## Outlook

The results presented in this thesis are promising for studying unique features of dipolar BECs, e.g. their roton-like excitation spectrum [28] and self-organized structures [24, 26]. The roton-maxon excitation spectrum predicted for a dipolar oblate BEC differs fundamentally from the spectrum of contact interacting systems. For a purely contact interacting BEC, the excitation spectrum increases monotonically with the momentum $q$ of the excitation. For small momenta, the spectrum exhibits a phonon-like behaviour, increasing linearly with $q$. For higher momenta, the spectrum increases quadratically as for a free particle (similar to the green dashed-dotted curve in figure 6.1a). In strong contrast, the excitation spectrum of an oblate dipolar BEC does not increase monotonically due to the momentum-dependent dipolar interaction and exhibits a local minimum ${ }^{1}$ called roton minimum (see figure 6.1a). This roton minimum defines a new characteristic length scale in the system, which is associated to self-organized structures [24].

In our setup using the one-dimensional optical lattice, we do not produce a single oblate dipolar BEC. Typically we create a stack of around $N_{l a t}=10-15$ oblate BECs coupled by inter-site interactions. This is predicted to modify the roton-maxon spectrum [30, 31]. For example, assuming a system with two sites ( $N_{\text {lat }}=2$ ), two different branches appear in the excitation spectrum, one mode stiffer and the other one softer compared to the case of a single dipolar BEC (shown in figure 6.1b). As a consequence, the roton minimum emerges even for larger scattering lengths, i.e. for negative scattering lengths with smaller absolute values. This 'hybridization' becomes even more pronounced in a stack of coupled

[^13]$\mathrm{BECs}^{2}$. Indeed, the excitation spectrum hybridizes in $N_{\text {lat }}$ modes, as one site couples with all other $N_{\text {lat }}-1$ sites. The more lattice sites are loaded, the more softened is the lowest lying roton-mode and the roton minimum is shifted to smaller momenta compared to the case of a single BEC (see figure 6.1c).

The calculations shown in figure 6.1 assume a homogeneous BEC with infinite radial extension. As our dipolar BECs are finite in size, the roton-maxon spectrum is not continuous anymore. However, as shown in [89], the roton minimum is still observable in a finite size BEC. Further simulations have to show whether the roton minimum is observable in our stack of dipolar BECs or not.

A way to probe the roton minimum could be to induce a 'roton instability', which emerges when the energy associated to the roton minimum is equal to zero. This roton instability - occurring at finite momentum - is fundamentally different from the 'phonon instability' observed in previous d-wave collapse measurements [20]. Therefore, the dynamics associated to the roton instability is expected to be significantly different. Hence, in the next step we would like to induce a roton instability by ramping fast over the stability threshold, similar to the experimental procedure presented in [20].

[^14]
## A. Appendix

## A.1. Two-Frequency Acousto-Optic Modulator Driver

(a)

(b)


Figure A.1.: (a) standard AOM driver setup to control the laser beam intensity with a voltage controlled oscillator (VCO) that creates the frequency $f_{1}$ (blue line) an its attenuator is controlled by the external voltage. This signal is amplified and goes to the AOM. (b) two-frequency AOM driver setup with an additional VCO with frequency $f_{2}$ that is added before the amplifier to keep a constant radio-frequency power on the AOM crystal constant. For this a control voltage dependent voltage adjustment is needed. Taken from [34].

An acousto-optic modulator (AOM) uses the acousto-optic effect to diffract light from a standing sound wave inside a crystal. The sound wave induces a periodic change in the refractive index of the crystal, therefore, generating a grating for an incoming light beam. The resulting diffraction pattern at the output of the AOM can be modified. Hence, when blocking all beams except the first diffraction order, the AOM can be used to control the intensity of a light beam.

Experimentally we use two AOM setups to control the optical dipole trap (ODT). The crossed ODT is produced by a 1076 nm CW Ytterbium fiber laser (VLR-100-F) operating at a power of 64 W . The beam is splitted in two parts: one corresponding to ODT1, the other one to ODT2. Each beam goes through an AOM and behind the AOMs, we measure a maximum power of 17.7 W for ODT1 and 16.0 W for ODT2 (but only 8 W are used). For a standard AOM driver setup (see figure A.1a), the diffracted laser beam can be


Figure A.2.: Reflectivity of the AOMs (Crystal Tec 3110-199) used for both optical dipole traps. Minima are at 98,110 and 122 MHz . Therefore, we chose $f_{1}=98 \mathrm{MHz}$ and $f_{2}=122 \mathrm{MHz}$.
deflected about 1 mrad when changing the radio-frequency ( RF ) power. The higher the RF power the higher the crystal temperature. This can induce changes on the diffraction index, therefore leading to movements of the diffracted beam.

To improve the pointing stability of the ODTs in our experiment, we use a two-frequency AOM driver setup (see figure A.1b). This setup was build up in 2007 and is described in detail in [34, 61], but it had to be renewed due to a damaged analog voltage control for the second attenuator. In the following we describe the new programming of the digital voltage control and the measured pointing stability.

## Reflectivity of the AOM

As the name of the driver suggests, we need two frequencies $\left(f_{1}\right.$ and $\left.f_{2}\right)$ on the AOM. To find the correct frequencies $f_{1}$ and $f_{2}$, the reflectivity of the used AOM (Crystal Tec 3110-199) had to be measured. First the input power after the amplifier to the AOM was measured with a spectrum analyzer. To measure the reflectivity, a coupler (2DC_10_1+) between the amplifier and the AOM is built in. The back reflected signal is measurable at the coupler 'output', where we measure the backreflected power again with a spectrum analyzer. The obtained reflectivity is shown in figure A. 2 with minima at 98,110 and 122 MHz . Therefore, the used frequencies for the two-frequency driver setup are $f_{1}=98$ MHz and $f_{2}=122 \mathrm{MHz}$.

## Digital Voltage Adjustment

The previous used voltage adjustment circuits were analog and had to be calibrated with six potentiometers. Our electric workshop built digital voltage adjustments, which are


Figure A.3.: Pointing stability of single frequency driver setup compared to a two frequency driver setup. The vertical movement against different ODT1 beam intensities. The single frequency driver shows a maximum vertical movement of 0.67 mrad . For the two-frequency driver setup it is reduced by a factor of around 20 to maximum 0.03 mrad .
working with a digital list, containing incoming control voltage and the corresponding output voltage. The outcoming voltage has to be chosen such that the resulting power of $f_{1}$ and $f_{2}$ on the AOM crystal is constant. We looked simultaneously at both frequency powers by using a spectrum analyzer after the last amplifier and programmed the outcoming voltage for the attenuator of frequency $f_{2}$. This was done for sufficient small steps of the incoming control voltage.

## Pointing Stability

After reinstalling the new AOM driver setup, we checked the pointing stability of the diffracted beam (first order of diffraction). We measured the ODT1 movement at a distance of $d=1.05 \pm 0.05 \mathrm{~m}$ after the AOM, for the single frequency driver setup and the two-frequencies driver setup. The intensity of the Gaussian laser beam (ODT1) was measured with a four quadrant photodiode to determine the position of the beam. The photodiode measures the intensity on four neighboring quadrants. We extract the sum of the signals detected on all quadrants $A$ and the horizontal and vertical differences $\Delta U_{v, h}$. The beam waist is measured to be 0.673 mm at the photodiode and the spatial beam movement is evaluated with Gaussian beam theory. The integrated 2D-Gauss beam with intensity $A^{\prime}$ yields to a measured power on all the four quadrants of

$$
\begin{equation*}
A \equiv \int_{-\infty}^{\infty} \mathrm{d} x \int_{-\infty}^{\infty} \mathrm{d} y A^{\prime} e^{-\frac{x^{2}}{2 \sigma^{2}}-\frac{y^{2}}{2 \sigma^{2}}}=A^{\prime} 2 \pi \sigma^{2} . \tag{A.1}
\end{equation*}
$$

The position of the quadrant photodiode was aligned to have $\Delta U_{v, h}$ for maximum laser intensity. The measurable difference signals $\Delta U$ for other laser intensities is resulting from the spatial beam movement $b$

$$
\begin{equation*}
\Delta U=\int_{-b}^{b} \mathrm{~d} x \int_{-\infty}^{\infty} \mathrm{d} y A^{\prime} e^{-\frac{x^{2}}{2 \sigma^{2}}-\frac{y^{2}}{2 \sigma^{2}}}=A^{\prime} \sqrt{2 \pi} \sigma \int_{-b}^{b} \mathrm{~d} x e^{-\frac{x^{2}}{2 \sigma^{2}}} \tag{A.2}
\end{equation*}
$$

with the substitution $\tau^{2}=\frac{x^{2}}{2 \sigma^{2}}$. Using that a Gaussian function is symmetric one gets

$$
\begin{equation*}
\Delta U=A^{\prime} \sqrt{2 \pi} \sigma \int_{-b / \sqrt{2} \sigma}^{b / \sqrt{2} \sigma} \mathrm{~d} \tau e^{-\tau^{2}} \sqrt{2} \sigma=A \operatorname{erf}\left(\frac{b}{\sqrt{2} \sigma}\right) \tag{A.3}
\end{equation*}
$$

with the error function $\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-\tau^{2}} \mathrm{~d} \tau$. As the inverse error function is not available in every calculations program, we determine $b$ by searching for the zero ${ }^{1}$ of

$$
\begin{equation*}
\Delta U-A \operatorname{erf}\left(\frac{b}{\sqrt{2} \sigma}\right)=0 \tag{A.4}
\end{equation*}
$$

which is only solvable for a single value of movement $b$. The angular movement is finally determined with the distance as $b / d$.

The measured angular movement in vertical direction is shown in figure (A.3). The single frequency driver shows a maximum movement of 0.67 mrad , whereas the twofrequencies driver reduces the maximum movement by a factor of 20 to 0.03 mrad . The horizontal movement of the beam is not shown on the figure. It has a maximum movement of 0.06 mrad for both driver setups.

## A.2. Written Programs for the Evaluation

As a part of my thesis I have written several programs for the evaluation of the absorption images of atomic clouds. I will give here a short overview on the three procedures included in the already existing automatic evaluation program. These programs were necessary for the evaluation of the stability of the BEC in the one-dimensional lattice.

## A.2.1. 1D Bimodal Fit

From the taken absorption images the optical densities of the atomic clouds are extracted with MatLab. The fitting procedures to extract the atom number, cloud size or temperature are done with an automatic evaluation program. In this already existing evaluation MatLab program, a new fitting procedure ${ }^{2}$ was included to extract the BEC atom number

[^15]

Figure A.4.: The integrated density distribution is plotted with black crosses. The Gaussian fit (dark blue line) is performed without the area inside the magenta lines (RONI). The parabolic function is fitted on the data after subtracting the blue plotted Gaussian fit (green crosses and turquoise line). The final bimodal fit, including parabolic and Gaussian shape, is shown in red.
for the stability of a BEC in the one-dimensional optical lattice. In this case the absorption images exhibit interference peaks. Therefore, to simplify the fitting procedure, each absorption image is integrated along the lattice direction.

The atom distribution density is already integrated in one direction due to the used absorption imaging. For this reason we have to integrate twice equation (2.18) describing the density distribution of a BEC after a TOF. Performing the first integration, it gives the two dimensional distribution [90, chapter 1.2.3.3]

$$
\begin{equation*}
n_{2 D}\left(y, z, t_{T O F}\right)=\frac{5 N}{2 \pi R_{y}\left(t_{T O F}\right) R_{z}\left(t_{T O F}\right)} \max \left[1-\left(\frac{y}{R_{y}\left(t_{T O F}\right)}\right)^{2}-\left(\frac{z}{R_{z}\left(t_{T O F}\right)}\right)^{2}, 0\right]^{3 / 2} \tag{A.5}
\end{equation*}
$$

with $R_{z}\left(t_{T O F}\right)$ the Thomas-Fermi radius after the expansion time $t_{T O F}$. After integrating also in a second direction (z) one get the following one-dimensional (1D) density distribution

$$
\begin{equation*}
n_{1 D}\left(y, t_{T O F}\right)=\frac{15}{16} \frac{N}{R_{y}\left(t_{T O F}\right)} \max \left[1-\left(\frac{y}{R_{y}\left(t_{T O F}\right)}\right)^{2}, 0\right]^{2} . \tag{A.6}
\end{equation*}
$$

However experimentally, the atomic clouds present a condensed BEC fraction and also a thermal fraction (not condensed). For a thermal cloud the integrated 1D distribution writes

$$
\begin{equation*}
n_{T, 1 D}\left(y, t_{T O F}\right)=\frac{N_{T}}{\sqrt{2 \pi} l_{y}\left(t_{T O F}\right)} e^{-\frac{y^{2}}{2 l_{y}^{2}}} . \tag{A.7}
\end{equation*}
$$



Figure A.5.: Typical diffraction pattern integrated perpendicular to the lattice direction (red line). For each 'positive' diffraction order on the right side, we extract the relative populations. This is done by using a multiple Gaussian fit with constant spacing (blue lines).

To extract the BEC atom number the recorded absorption image is fitted in the following way. We integrate the two dimensional density distribution along the lattice direction to get the 1D distribution. This consists of a 1D parabolic BEC fraction and a 1D Gaussian shaped thermal fraction, shown with black crosses in figure A.4. First we fit a Gaussian function to the 1D data to have good starting parameters for the following fits. We then define a region of no interest (RONI) where all BEC atoms sit and we remove this region from the 1D raw data (two magenta lines). Then we fit a second Gaussian function to this modified distribution to extract the maximum thermal density (dark blue line).

For the next fitting procedure, we subtract the fitted thermal density distribution from the 1D raw data (green in figure A.4). Then we fit the 1D parabolic density distribution (turquoise line) on this remaining density distribution. After these fitting procedures the maximum BEC density and the maximum thermal density are fixed. Finally on the whole 1D data, a bimodal fit including a Gaussian function and the parabola is fitted with fixed maximum densities (red line). We extract the BEC radius from which we determine the condensed atom number using equation (A.6).

## A.2.2. Raman Nath Evaluation

The corresponding program ${ }^{3}$ is used to calibrate the lattice depth via the diffraction of a BEC from a pulsed standing light wave (see section 4.4.2 and 4.4.2.2). Before running the program with 'evaluateRamanNath' one has to choose a picture with many visible orders and mark two regions in the chosen picture. First the zeroth order with 'New

[^16]ROI for RamanNath0' and the last visible order with 'New ROI for RamanNath1'. After starting the program, we determine first the distance between these two orders. We fit the center position of both orders by using two single Gaussians. The spatial difference of these orders has to be a multiple of the characteristic neighboring spacing $\Delta z$ (4.18). Therefore, we calculate the maximum number of diffraction orders $n_{\max }$ by calculating the nearest multiple of $\Delta z$. After this precalculations, the real procedure to extract the relative populations of the orders starts.
For each diffraction pattern, measured for different lattice pulse durations $t_{\text {pulse }}$, we extract the population dynamics of all diffraction orders by following procedure:

1. The diffraction pattern is integrated along the $y$-direction to get the data similar to figure A.5.
2. The program searches for the center of the diffraction pattern. For this the data is averaged over 50 pixels to receive a very smoothen picture. On this smoothen picture we fit a Gaussian function to extract the center position $z_{0}^{\prime}$.
3. We fit in the region $\left[z_{0}^{\prime}-\Delta z / 2, z_{0}^{\prime}+\Delta z / 2\right]$ a second Gaussian function to find the real center $z_{0}$ of the zeroth order.
4. We fit $n_{\max }+1$ Gaussian functions with spacing $\Delta z$ on the right 'positive' diffraction orders (blue lines in figure A.5). From the height and the widths, we can extract the atom number using equation A.7.

Experimentally the diffraction patterns are sometimes asymmetric and the first centering step fails. Because of this the program asks after the fourth step if the center position $z_{0}$ needs to be corrected. This can be done in units of half characteristic spacings $\Delta z$ by typing it in the command line. An empty line continues the fitting procedure with the next image.

When all images are evaluated, another procedure ('lifecalcRamanNathNew_02.m') starts automatically. For every extracted population dynamics of the $n_{\max }+1$ orders, we perform an independent fit with the theoretical populations given in equation (4.38). So the next described steps are $n_{\max }+1$ times performed. As the function (4.38) is only numerically available, a standard fitting procedure is not possible. Instead, we determine roughly the lattice depth via $s=4 n_{\max }^{2}$ and calculate the functions for many lattice depths $V_{0}$ around this value. In the next step, we calculate for all functions the sum of squared residuals (difference between the population dynamics and the function) compared to the experimental relative populations. In the end we take the function with the smallest difference (least square method) to find the best lattice depth $V_{0}$ describing the experimental population dynamics.

The final outputs are figures like 4.6 to determine the lattice depth and the program creates automatically figures like 4.5 .

(a) Modified experimental image

(b) Fitted ellipse to the data

Figure A.6.: (a) Modified absorption image. All data points with optical density higher than 0.02 are set to unity (red-brown), otherwise to zero (blue). (b) A two-dimensional structure with unity inside an ellipse and otherwise zero, is fitted on a to determine the center position.

## A.2.3. Averaging of Interference Pictures

The corresponding program ${ }^{4}$ is used for the calibration of the lattice depth via the interference patterns and the relative side-peak population (see sections 4.4.1). The fitting procedure is very similar to our 'standard' 2D bimodal fit 'cloudfitgauss2droni_3.m' (it has the same fit procedure in 2D, as the described fitting procedure ' 1 D bimodal fit' in section A.2.1). Nevertheless we point out the procedure to center the images by using the thermal cloud distribution. To fit only the thermal density distribution, we have to remove the region where all BEC atoms sit. As the interference patterns are distributed over the whole thermal cloud, there are only a few data points left to fit the thermal distribution. Therefore, the uncertainty on the corresponding center of the thermal cloud is large.

To overcome this problem, we use the following method: we set to unity all the data points with optical density (OD) higher than typical 0.02 and for $\mathrm{OD}<0.02$ we set the data points to zero, as shown in figure A.6a. As a result we see an elliptic shape which we fit a 2 D structure with the value unity inside

$$
\begin{equation*}
f(y, z)=\frac{\left(y-y_{0}\right)^{2}}{l_{y}^{2}}+\frac{\left(z-z_{0}\right)^{2}}{l_{z}^{2}} \tag{A.8}
\end{equation*}
$$

and otherwise zero, shown in figure A.6b. As a result of this, we extract exactly the center of the thermal cloud by the fitting parameters $y_{0}$ and $z_{0}$. With this determined center of all images, once they have been shifted accordingly to their center. The final outputs of the program are averaged pictures like figure 4.4a.

[^17]
## A.3. Roton-Maxon Excitation Spectrum

## Roton-Maxon Spectrum for a single two-dimensional Dipolar Condensate

The dispersion relation of a purely contact interacting BEC is the Bogoliubov excitation spectrum of elementary excitations

$$
\begin{equation*}
E(q)=\sqrt{E_{q}\left(E_{q}+2 \mu_{\text {contact }}\right)} \tag{A.9}
\end{equation*}
$$

with $q$ the quasi momentum, $E_{q}=\frac{\hbar^{2} q^{2}}{2 m}$ the free particle energy and $\mu_{\text {contact }}$ the chemical potential as in equation (2.20). For small $q$ the dispersion relation is phonon-like $E(q)=$ $\frac{\mu \hbar}{2 m} q$ and for higher momenta, it is like the one of a free particle $E(q)=\frac{\hbar^{2} q^{2}}{2 m}$. The instability occurs here for $q=0$ and is called phonon instability.

However, for a dipolar BEC, the excitation spectrum is different due to the momentumdependent dipolar interaction. In particular, the excitation spectrum of a two-dimensional homogeneous dipolar BEC, infinitely elongated in the radial directions and strongly confined in $z$-direction, was predicted to have a minimum, called roton minimum, and a maximum [28]. Assuming that the wavefunction of the BEC in z-direction is exponential decaying, the excitation spectrum writes [30]

$$
\begin{equation*}
E(q)=\sqrt{E_{q}\left(E_{q}+2 A\right)} \tag{A.10}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\mu-\mu_{\text {dipole }}(q)=\mu-2 g_{d d} n_{0} F\left(\frac{q l_{z}}{\sqrt{2}}\right) \quad \text { with } \quad F(x)=\frac{3 \sqrt{\pi}}{2}|x| \operatorname{erfc}(x) e^{x^{2}} \tag{A.11}
\end{equation*}
$$

with $\operatorname{erfc}(x)=1-\operatorname{erf}(x)$ the complementary error function, $\mu=\left(g+2 g_{d d}\right) n_{0}$ is the chemical potential of the dipolar BEC with $n_{0}$ the maximum density and $l_{z}=\sqrt{\hbar / m \omega_{l a t}}$ is the harmonic oscillator length associated to the BEC wavefunction in the lattice direction.

The resulting dispersion relation exhibits a roton minimum for proper values of $g$. In particular in figure 6.1a is shown the dispersion relation for our experimental parameters. We observe a roton minimum in the dispersion curve only for scattering length close to the instability point, i.e. the point at which the energy associated to the roton minimum is equal to zero. This instability is different from the phonon instability ${ }^{5}$, because it occurs at finite momentum. It is called the roton instability. One can finally note that the regime where the roton minimum appears in the dispersion curve, is as small as $1 a_{0}$ for our parameters, which makes it difficult to observe experimentally.

## Two coupled Dipolar BEC

For two interacting dipolar BECs, inter-site coupling due to the long-range dipolar interaction, leads to a hybridization [30] of the dispersion relation in two continuous curves

[^18]with a stiffer ( + ) and a softer (-) mode. The dispersion relation for two identical dipolar BECs is given by
\[

$$
\begin{equation*}
E_{ \pm}(q)=\sqrt{E_{q}\left(E_{q}+2 A \pm 2 C\left(d_{l a t}\right)\right)} \tag{A.12}
\end{equation*}
$$

\]

where

$$
\begin{equation*}
C\left(d_{l a t}\right)=\mu_{\text {inter }}\left(d_{l a t}\right)-\mu_{\text {inter }}\left(q, d_{l a t}\right)=2 g_{d d} n_{0} e^{-d_{\text {lat }}^{2} / 2 l_{z}^{2}}-2 g_{d d} n_{0} \tilde{F}\left(\frac{q l_{z}}{\sqrt{2}}, \frac{d_{l a t}}{\sqrt{2} l_{z}}\right) \tag{A.13}
\end{equation*}
$$

with

$$
\begin{equation*}
\tilde{F}=\frac{3 \sqrt{\pi} x e^{x^{2}}}{4}\left(e^{-2 x y} \operatorname{erfc}(x-y)+e^{2 x y} \operatorname{erfc}(x+y)\right) \tag{A.14}
\end{equation*}
$$

Note that the roton minimum is observable at higher (less negative) scattering length compared to a single dipolar BEC due to the softening of one mode.

## Stack of $N$ coupled Dipolar BEC

For a stack of $N$ interacting two dimensional BEC (created with a one-dimensional optical lattice), the hybridization becomes even more pronounced. A plane-wave ansatz leads to coupled Bogoliubov-de Gennes equations for $f_{q, i}=u_{q, i}+v_{q, i}$ and the $N$ continuous dispersion relations are given as the eigenvalues of a system of $N$ linear equations

$$
\left(\begin{array}{ccccc}
E_{q}\left(E_{q}+2 A\right) & 2 E_{q} C\left(d_{l a t}\right) & 2 E_{q} C\left(2 d_{l a t}\right) & \cdots & 2 E_{q} C\left(N d_{l a t}\right)  \tag{A.15}\\
2 q_{q} C\left(d_{l a t}\right) & E_{q}\left(E_{q}+2 A\right) & 2 E_{q}\left(\left(d_{l a t}\right)\right. & & \left.2 E_{q} C(N-1) d_{a t}\right) \\
2 E_{q} C\left(2 d_{l a t}\right) & 2 E_{q} C\left(d_{l a t}\right) & E_{q}\left(E_{q}+2 A\right) & & 2 E_{q} C\left((N-2) d_{l a t}\right) \\
\vdots & & & \ddots & \vdots \\
2 E_{q} C\left(N d_{l a t}\right) & 2 E_{q} C\left((N-1) d_{l a t}\right) & 2 E_{q} C\left((N-2) d_{l a t}\right) & \cdots & E_{q}\left(E_{q}+2 A\right)
\end{array}\right)\left(\begin{array}{c}
f_{q, 1} \\
f_{q, 2} \\
f_{q, 3} \\
\vdots \\
f_{q, N}
\end{array}\right)=E^{2}\left(\begin{array}{c}
f_{q, 1} \\
f_{q, 2} \\
f_{q, 3} \\
\vdots \\
f_{q, N}
\end{array}\right) .
$$

Calculating the $N$ eigenvalues for several $q$ gives the hybridization for the dispersion relation.

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[^0]:    ${ }^{1}$ Alkali atoms have one valence electron therefore a magnetic moment of one Bohr magneton
    ${ }^{2}$ the dipole strength is 36 times larger for chromium than for alkali atoms

[^1]:    ${ }^{3}$ in the experimental section 3 the atom numbers are for chromium BEC around $N=20,000$

[^2]:    ${ }^{4}$ This special trap is used for the calibration of the scattering length in section 3.2.1

[^3]:    ${ }^{1}$ the one-dimensional lattice, used for the stability measurements in chapter 5 , is entirely specified in chapter 4

[^4]:    ${ }^{2}$ Experimentally we are limited to 35 G above the Feshbach resonance. There the scattering length is around $97 a_{0}$

[^5]:    ${ }^{1}$ if the polarizations are not in the same plane one would have a polarization gradient with circularly and linearly polarized light.

[^6]:    ${ }^{2}$ IPG: 'YLR-20-1064-LP-SF'
    ${ }^{3} \mathrm{TeO}_{2}$-S modulator, model MTS80

[^7]:    ${ }^{4}$ Note that our rise time of the used AOM is $1 \mu \mathrm{~s}$. But the effective pulse length (integration over the pulse) matches quite well with the pulse duration $t_{\text {pulse }}$.

[^8]:    ${ }^{5}$ Mathematica: MathieuC $\left[a_{r}, q, x\right]$ gives the even Mathieu functions for known $q$ and $a_{r}$

[^9]:    ${ }^{6}$ Mathematica: MathieuCharacteristicA $[r, q]$ : gives the characteristic value $a_{r}$ for even Mathieu functions

[^10]:    ${ }^{7}$ unitless momentum $k$ due to unitless space parameter $z=k_{l a t} x$

[^11]:    ${ }^{1} V_{0}(t)=V_{0} \cdot\left((k+1) t^{k}-k t^{k+1}\right)$ with $k=11$ and $t$ is linearly increased from zero to one within the ramping time

[^12]:    ${ }^{2} N=\max \left[N_{0}\left(a-a_{\text {crit }}\right)^{\beta}, 0\right]$ where $N_{0}, a_{\text {crit }}$ and $\beta$ are fitting parameters

[^13]:    ${ }^{1}$ The excitation spectra presented in figure 6.1 are calculated with the formulae in [30] (see appendix A.3)

[^14]:    ${ }^{2}$ In addition, the presence of inter-site coupling in a stack of dipolar BECs is responsible for the appearance of self-organized structures, which would not be present in a single BEC [32, 88].

[^15]:    ${ }^{1}$ MatLab: command 'fzero'
    ${ }^{2}$ ep.traptype 'LatticeStability' - ip.plottype 'LatticeStability' - program 'cloudfitgauss1droni.m'

[^16]:    ${ }^{3}$ Runs with 'evaluateRamanNath' - ep.traptype 'RamanNathDiffraction' - program 'cloudfitRamanNathNew.m' and 'lifecalcRamanNathNew_02.m'

[^17]:    ${ }^{4}$ ep.traptype 'LatticeCenterPic' - program 'cloudfitEllipse.m' and 'lifecalcCenterPic.m'

[^18]:    ${ }^{5}$ a phonon instability can still occur in a dipolar BEC if the roton minimum does not 'touch the zero'

