Exercise 1: s- and d-wave Scattering of ultracold $^{87}$Rb Atoms

The two following papers discuss scattering properties of ultracold $^{87}$Rb atoms.


Read both papers and answer the following questions:

a) How was the ultracold atomic cloud prepared? How was it split, separated and made to collide? How large were the typical collision speeds in m/s?

b) Why is there no p-wave scattering? What are singlet and triplet potentials?

c) How high is the centrifugal barrier for a d-wave collision measured in K? What speed in the center-of-mass is necessary to overcome the barrier?

Hint: For the molecular potential consider only the van-der-Waals term with $C_6 = 4707$ au.

d) What is the Newton sphere? Why does the s-wave scattering in the absorption images ideally produces a ring shape?

e) How does interference occur between s- and d- partial waves? How are the scattering cross-sections $\sigma_s$ and $\sigma_d$ determined from the absorption images?

f) What is a shape resonance?
Exercise 2: Evaporative Cooling

Evaporative cooling is used to cool atoms below temperatures obtained with laser cooling. A thermal atomic ensemble in a magnetic trap can be cooled by applying a radio-frequency field to transfer atoms to non-trapped Zeeman states.

\[
\frac{\hbar}{c} \frac{1}{110} E \times \begin{array}{c}
m_f = +2 \\
m_f = +1 \\
m_f = 0 \\
m_f = -1 \\
m_f = -2 \\
m_f = +2 \\
\end{array}
\]

\[ -x_{\text{cut}} \quad +x_{\text{cut}} \]

a) Why is the ensemble temperature decreased by removing atoms at certain positions \( \pm x_{\text{cut}} \) in space? Explain the mechanism.

b) Consider a cloud of atoms at \( T = 50 \mu K \) in a harmonic potential with a trap frequency of \( \omega/2\pi = 100 \text{ Hz} \). What is the necessary radio-frequency \( \nu \) to remove 10% of the atoms?

c) In a thermal ensemble of \(^{87}\text{Rb}\) atoms the velocity distribution of atoms is given by the Maxwell-Boltzmann formula

\[
F(v) = \sqrt{\frac{2}{\pi}} \left( \frac{m}{k_B T} \right)^{3/2} v^2 e^{-\frac{mv^2}{2k_BT}}.
\]

Calculate the new equilibrium temperature after removing 10% of the fastest atoms.  
*Hint:* The total energy is conserved during rethermalization.

d) After reaching thermal equilibrium another 10% of the initial atom number is removed. What is the new equilibrium temperature? Compare to the temperature acquired after a single cooling step with 20% of the fastest atoms removed. How would you choose the evaporation steps in order to increase the cooling efficiency in a real experiment?