Exercise 1: Magneto-optical trap (MOT) 30(4,6,8,6,6) points

a) Describe the working principle of a MOT considering a 1D model with a single atom.

b) The force for an atom in the MOT is given by

\[ F_{\text{MOT}}(v, z) = -\beta v - \kappa z \]

with the coefficients

\[ \beta = \frac{8\hbar k^2 s_0 \delta / \Gamma}{(1 + s_0 + 4(\delta / \Gamma)^2)^2} \]

\[ \kappa = \frac{g_j \mu_B B'}{\hbar k} \beta. \]

Determine the numerical values of \( \beta \), \( \beta/M \) and \( \sqrt{\kappa/M} \) for the \( 5S_{1/2} \rightarrow 5P_{3/2} \) transition in \(^{87}\text{Rb}\) (\( M \) is the atomic mass). What do these numbers mean? Use the following parameters: \( B' = 10 \text{G/cm}, s_0 = I/I_{\text{sat}} = 1 \) and \( \delta / \Gamma = 1 \).

Hint: Relevant quantities for Rubidium can be found in http://steck.us/alkalidata/

c) Give the equation of motion for an atom in the MOT. What is the total energy of the atom?

Consider an ensemble of atoms, its energy distribution is given by the Maxwell-Boltzmann distribution \( f(E) \propto e^{-E/k_B T} \). Show that the phase-space (position and velocity) distribution can be written as:

\[ n(z, v, T) = n_0 \cdot e^{-z^2/2\sigma_z^2} \cdot e^{-v^2/2\sigma_v^2}. \]

d) The spatial capture range of a MOT is limited by the surface on which the Zeeman shift compensates for the laser detuning \( \delta \). Calculate the capture radius \( z_c \) for \(^{87}\text{Rb}\) using the MOT parameters from the previous exercise.

Calculate the maximum speed \( v_c \) that a \(^{87}\text{Rb}\) atom can have to be captured by this MOT. To simplify the calculation, assume that an atom is pushed with the maximum force \( F_{\text{max}} = \frac{\hbar k \Gamma}{2} \) in the entire MOT region.

Note: Using Mathematica it is also not difficult to calculate the maximum capture velocity for the exact force \( F_{\text{MOT}} \).

e) The capture rate of atoms into a MOT is constant while the loss rate is proportional to the number of atoms in the MOT. Write the rate equation that describes the number of atom in the MOT. As capture rate use \( R \) [Atoms/s] and for loss rate consider \( \gamma \) [1/s]. Solve the ordinary
differential equation and sketch the solution for different values of $R$ and $\gamma$.

*Hint:* Orders of magnitude $10^7 / s < R < 10^{10} / s$ and $10^{-2} / s < \gamma < 10 / s$.

**Exercise 2: MOT Temperature** 20(12,4,4) points

In addition to the maximum number of trapped atoms another important parameter of a MOT is the temperature reached $T$ of the atomic cloud. To experimentally determine this temperature, the free expansion of the atomic cloud is observed after switching off the trap. Assuming a non-interacting ensemble, the atoms move apart freely. The temperature determines the width of the original velocity distribution of the atoms, so from the time evolution of the density distribution the temperature can be determined.

a) Determine the atomic density distribution $n_z(z,t)$ at time $t$.

*Hint:* This can be found from the phase-space distribution obtained in exercise 1c by applying the transformation $z = z' + \nu't$ and integrating over all velocities.

b) Plot the temporal behavior of the spatial width of the cloud.

c) How does $\sigma_z(t)$ behave for large times or small initial width $\sigma_z(t = 0)$? What is the easiest way to measure the temperature of the atomic cloud?

*Note:* The first MOT was made by Raab et al.\(^1\) You can find a discussion of the MOT by Townsend et. al.\(^2\).
